Assessing Optimal CO₂ Abatement Policies for the Kyoto Protocol:
A Genetic Algorithms Approach

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Abstract

In 1997, the third Conference of the Parties (COP3) to the United Nations Framework Convention on Climate Change was held. Commitments were set for reducing greenhouse gas emissions in developed countries. Several models have been developed in order to analyze CO₂ abatement policies. These models should be categorized as global models considering the wide scale of global warming. Some of those models, however, analyze the policies on a one-country basis and models of global content divide the world into certain regions. It is not appropriate to implement the same policies to a region. A multi-country model is preferable to such models. Thus, we have constructed a macroeconometric model linked with an energy model to assess CO₂ abatement policies applying genetic algorithms to quantify the optimal policy in favor of the Kyoto Protocol.

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1. Introduction

Global warming is one of the important world policy issues. In 1997, the Third Conference of the Parties (COP3) to the United Nations Framework Convention on Climate Change (UNFCCC) was held. Commitments were set for reducing greenhouse gases (GHGs) relative to the 1990 level in developed countries. The Kyoto Protocol shows that developed countries as a whole must show a reduction of -5.2 percent between the years 2008 and 2012. As for major industrial countries, commitments are -8.0 percent for the European Union (EU), -7.0 percent for the United States, -6.0 percent for Canada and Japan and +8.0 percent for Australia. In the EU, the targets for Germany, the United Kingdom, Italy and France are at -21.0 percent, -12.5 percent, 0.0 percent and -6.5 percent of GHGs, respectively.

In order to form and evaluate policies pertaining to the reduction of GHGs, several models have been developed. In terms of their objectives, there are global models (e.g. Burniaux, Nicoletti and Oliveira-Martins 1992; Duchin and Lange 1994; Duraiappah 1993; Edmonds and Reilly 1983; Lu and Kaya 1988; Manne and Richels 1992; Manne, Mendelsohn and Richels 1995; Matsuoka, Kainuma and Morita 1995; Nordhaus 1994, 2000) and domestic models (e.g. Jorgenson and Wilcoxen 1990, 1993a, 1993b; Proops, Faber and Wagenhals 1993). It is true that these models show certain results on analyzing global warming, but it is also a fact that they hold a problem. Global models are preferable to domestic models, as global warming is a world issue; however, global models treat the world as one country or divide the world into several regions. It is not appropriate to implement the same policies to a region. Hence, we need to analyze global warming in a multi-country context. In this paper, we have constructed macroeconomic models linked with energy models that cover Japan, the United States, Canada, Germany, the United Kingdom, France, Italy and Australia. By using this model, we have assessed CO₂ abatement policies applying genetic algorithms to quantify the optimal policy in favor of the Kyoto Protocol.²

This paper is organized as follows. Section 2 discusses the theory of an optimal policy. Section 3 shows the methodology of genetic algorithms. Section 4 presents the model structure. Section 5 provides a preliminary estimation result of the US monetary policy.

2. Theory of Optimal Policy

Social Welfare Function and Policy Reaction Function

A social welfare function consists of a nation’s policy targets (e.g. economic growth, inflation rate,
government deficits and so forth). This indicates the course of economic policies. We usually formulate a
social welfare function in a quadratic form as:

\[
F = \sum_i w_i (Y_i - Y_i^*)^2 + \sum_j w_j (X_j - X_j^*)^2 ,
\]

where \(w_i\) is the weight assigned to \(Y_i\), \(Y_i\) is the \(i\)th policy objective variable, \(Y_i^*\) is the desired value of the
\(i\)th policy objective variables, \(w_j\) is the weight assigned to \(X_j\), \(X_j\) is the \(j\)th policy instrument and \(X_j^*\) is
the desired value of the \(j\)th policy instrument. We obtain the optimal policy by minimizing equation (1)
with respect to a policy instrument subject to an econometric model.

Here we show the case of two policy objective variables and two policy instruments. In this case we
can write a social welfare function as follows:

\[
F = w_1 (Y_1 - Y_1^*)^2 + w_2 (Y_2 - Y_2^*)^2 + w_{11} (X_1 - X_1^*)^2 + w_{22} (X_2 - X_2^*)^2 .
\]

Minimizing equation (2) by \(X_1\) subject to an econometric model yields

\[
2w_1 (Y_1 - Y_1^*) \left( \frac{\partial Y_1}{\partial X_1} \right) + 2w_2 (Y_2 - Y_2^*) \left( \frac{\partial Y_2}{\partial X_1} \right) + 2w_{11} (X_1 - X_1^*) + 2w_{22} (X_2 - X_2^*) \left( \frac{\partial X_2}{\partial X_1} \right) = 0.
\]

We suppose that conjectural variations between policy instruments are equal to zero, hence, the optimal
policy on \(X_1\) can be obtained as:

\[
X_1 = X_1^* - \left( \frac{w_1}{w_{11}} \right) (Y_1 - Y_1^*) \left( \frac{\partial Y_1}{\partial X_1} \right) - \left( \frac{w_2}{w_{11}} \right) (Y_2 - Y_2^*) \left( \frac{\partial Y_2}{\partial X_1} \right).
\]

Based on a similar methodology, Pissarides (1972) and Friedlaender (1973) analyzed the British and
the U.S. macroeconomic policies, respectively.
3. Theory and Application of Genetic Algorithms

Theory of Genetic Algorithms

Holland (1975) developed genetic algorithms which are based on biological evolutions. We can use them as one of the optimization techniques. They are composed of the following five steps: initialization, evaluation, selection, crossover and mutation. The computation of genetic algorithms is iterated from the second step to the fifth step until the fitness value reaches a certain criteria of the convergence. Detailed explanations of the five steps are as follows:

1. Step 1 (Initialization): To create the initial population of \(N\) chromosomes randomly. Each chromosome is a binary string of \(K\) bits. We generate random number \(r\) between 0.0 and 1.0. If \(r \geq 0.5\) or \(< 0.5\), the bit becomes 1 or 0, respectively. This process is repeated \(N \cdot K\) times.

2. Step 2 (Evaluation): To evaluate the fitness for each chromosome by using a fitness value function.

3. Step 3 (Selection): To select a new population by using a selection method. In this paper, we use the roulette wheel selection method. This method is based on the natural selection mechanism. The sum of fitness values \((F)\) can be written as:

\[
F = \sum_{i=1}^{N} f_i, \quad (x)
\]

where \(f_i\) = the fitness value of the \(i\)th chromosome. The probability of selection for the \(i\)th chromosome \((p_i)\) can be obtained as:

\[
p_i = \frac{f_i}{F}, \quad i = 1, 2, \ldots, N. \quad (x)
\]

Next, the cumulative probability for the \(i\)th chromosome \((q_i)\) can be written as:

\[
q_i = \sum_{j=i}^{N} p_j, \quad i = 1, 2, \ldots, N. \quad (x)
\]

We generate random numbers and if \(q_i < r < q_{i+1}\) \((i = 0, 1, \ldots, N - 1, q_0 = 0)\) satisfies, the \((i + 1)\)th
chromosome is selected. We repeat this process \( N \) times. Hence, \( N \) chromosomes are selected as a new population.

4. Step 4 (Crossover): To create offspring. In order to create offspring, we select pairs of parent-chromosomes randomly. We set the probability of crossover \( (p_c) \) and generate random numbers from the range between 0 and 1. If \( r_i < p_c \), the \( i \)th chromosome is selected as one of the parent-chromosomes. This procedure is repeated \( N \) times. Once parent-chromosomes are selected, we create offspring. We generate random integer numbers that are between 1 and \( K - 1 \). These random integer numbers are crossover positions. We replace bits after a crossover position between the parent-chromosomes. These new chromosomes (offspring) are added to, where parent-chromosomes are deleted from, the current population.

5. Step 5 (Mutation): To create new chromosomes which do not exist in the current population. We generate random numbers from the range between 0 and 1. If \( r < p_m \) (the probability of mutation), we change the bit from 0 to 1 or 1 to 0. This procedure is applied to each bit.

Application of Genetic Algorithms to Optimal Policy

Genetic algorithms are one of the optimization methods. Hence, we can apply them to empirical analyses on an optimal policy.

In most cases, in order to estimate a policy reaction function, we minimize a quadratic loss function subject to a macroeconometric model. Next, the policy reaction function is fed into the macroeconometric model. We can simulate the economy under the optimal policy by using that system.\(^4\)

Instead of estimating a policy reaction function, we use it as a fitness value function in genetic algorithms. Hence, genetic algorithms determine all parameters of a policy reaction function subject to a macroeconometric model. We note that parameters of a policy reaction function explained by genetic algorithms are not fixed.

4. The Model Structure

Our multi-country model consists of a macroeconomic block, a trade block and an energy block. The macroeconomic block is constructed on the basis of the Klein’s (1983) skeleton model. Eight countries’ macroeconomic blocks are linked to one other by the constant value share trade model. The energy block explains the final energy demand and \( CO_2 \) emissions using the real GNP that is determined in the macroeconomic block. The final energy price is also linked to explain the general prices in the
macroeconomic block. Here, we show the structure of the energy block.\textsuperscript{5}

Our energy block analyzes CO\textsubscript{2} emissions of G7 members and Australia. In this study, we estimated parameters by using panel data that combines eight countries' cross-sections and time series data between the years 1991 and 1996. We treat energy as one of production inputs and estimate the parameters. This model is a small econometric model that has twelve endogenous and five exogenous variables.

Our model has three major sub-blocks: i) final energy demand and CO\textsubscript{2} sub-block ii) prices sub-block iii) decomposition sub-block. We divide energy into coal, natural gas, oil and non-fossil fuels. The final energy demand and CO\textsubscript{2} sub-block explains these four energy demands and the amount of CO\textsubscript{2} emissions. In the prices sub-block, world coal and oil prices determine domestic coal, natural gas and oil prices. Some indicators on CO\textsubscript{2} emissions and the decomposition model are shown in the decomposition sub-block.

**Final Energy Demand and CO\textsubscript{2}Sub-block**

CO\textsubscript{2} emissions originate in the fossil fuel consumption and the use of fossil fuels is necessary in economic activities. This means that economic growth is one of the fundamental causes of global warming. Taking these factors into consideration, we derive at the final energy demand function from a two-level CES (Constant Elasticity of Substitution) production function whose inputs are capital stock, labor and energy. This two-level CES production function can be written as:\textsuperscript{6}

\[
Z_t = A_t \left[ a_1 E_{ft}^{b_1} + (1 - a_1) H_t^{b_1} \right]^{1/(1 - a_1)} \quad (8)
\]

\[
H_t = \left[ a_2 K_t^{b_2} + (1 - a_2) E_{ft}^{b_2} \right]^{1/b_2} \quad (9)
\]

\[
0 < a_1, a_2 < 1 \quad b_1, b_2 < 1
\]

where \(E_{ft}\) = final energy demand, \(A_t\) = the efficiency parameter, \(a_1\) and \(a_2\) = distribution parameters, and both \(b_1\) and \(b_2\) = substitution parameters. By marginal-product conditions, we can derive at the following equation from the two-level CES production function:

\[
\frac{K_t}{E_{ft}} = \left[ \frac{(1 - a_2)}{a_2} \right]^{1/(b_2 - 1)} \left[ \frac{P_t}{K_t} \right]^{1/(b_2 - 1)} \quad (10)
\]
where $P_{ki}$ is the user cost of capital and $P_{f}$ is the final energy price. This equation explains the final energy demand function. As for transformation losses from the primary energy to the final energy, the primary energy is shown as a function of the final energy demand. The primary energy supply function is:

$$E_{t} = f\left(E_{f}\right)$$  \hspace{1cm} (11)

where $E_{t}$ is the primary energy supply.

We can also define the primary energy supply as the summation of coal demand, natural gas demand, oil demand and non-fossil fuel demand. This equation can be written as:

$$E_{t} = E_{c} + E_{g} + E_{o} + E_{n}$$  \hspace{1cm} (12)

where $E_{c}$ is the coal demand, $E_{g}$ is the natural gas demand, $E_{o}$ is the oil demand and $E_{n}$ is the non-fossil fuel demand. These four energy shares determine their demands at the primary energy level. Each energy demand can be written as follows:

$$E_{c} = s_{c} E_{t}$$  \hspace{1cm} (13)

$$E_{o} = s_{o} E_{t}$$  \hspace{1cm} (14)

$$E_{n} = s_{n} E_{t}$$  \hspace{1cm} (15)

where $s_{c}$ is the natural gas share, $s_{o}$ is the oil share and $s_{n}$ is the non-fossil fuel share. The coal demand is determined as the residual:

$$E_{c} = E_{t} - E_{o} - E_{g} - E_{n}.$$  \hspace{1cm} (16)

Natural gas and oil shares are functions of relative energy price and their lagged values. These functions can be written as:
\[
\ln(s_{it}) = f \left[ \ln(s_{it-1}) \ln \left( \frac{P_{it}}{P_{it-1}} \right) \right]
\]  
\tag{17}

where \( i = C \) (coal), \( G \) (natural gas) and \( O \) (oil), \( j, k \neq i \), and \( k \neq j \) with \( P_C \) = the domestic coal price, \( P_G \) = the domestic natural gas price and \( P_O \) = the domestic oil price. The non-fossil fuel share is one of the policy instruments and the coal share is residual of the others in this model.

Each energy source has each CO\(_2\) emission coefficient. CO\(_2\) emissions released from coal, natural gas and oil consumption can be estimated as the following equation:

\[
CO_{2t} = R_C E_{Ct} + R_G E_{Gt} + R_O E_{Ot}
\]  
\tag{18}

where \( CO_{2t} \) = CO\(_2\) emissions, \( R_C \) = the CO\(_2\) emission coefficient of coal, \( R_G \) = the CO\(_2\) emission coefficient of natural gas and \( R_O \) = the CO\(_2\) emission coefficient of oil. In this paper we assume that \( R_C, R_G, R_O \) equal 1.08 carbon ton per ton of oil equivalent (toe) of coal, 0.62 carbon ton per toe of natural gas and 0.86 carbon ton per toe of oil respectively.

**Prices Sub-block**

This block explains domestic prices of coal, natural gas and oil, the average price of the primary energy and the final energy price. The three domestic energy prices depend on world coal price, world oil price and the exchange rate. World coal and oil markets determine their world prices so world coal and oil prices are exogenous variables in this model. Domestic coal, natural gas and oil prices can be written as:

\[
P_{Ct} = f_C (P_{coal,t} e_t)
\]  
\tag{19}

\[
P_{Gt} = f_G (P_{oil,t} e_t)
\]  
\tag{20}

\[
P_{Ot} = f_O (P_{oil,t} e_t)
\]  
\tag{21}

where \( P_{coal,t} \) = the world coal price in the US dollar and \( P_{oil,t} \) = the world oil price in the US dollar. As for imposing carbon taxes on the primary energy, prices of coal, natural gas and oil are reformulated as:
where $\tau_C = \text{the carbon tax on coal}$, $\tau_G = \text{the carbon tax on natural gas}$ and $\tau_O = \text{the carbon tax on oil}$. Next, we define the average price of the primary energy as the weighted average of domestic coal, natural gas and oil prices. This can be written as:

$$P_{Pt} = s_{Ct} P_{Ct} + s_{Gt} P_{Gt} + s_{Ot} P_{Ot}$$

(22)

where $P_{Pt} = \text{the average price of the primary energy}$. Finally, we assume that the final energy price is explained by the average price of the primary energy. The equation of the final energy price can be written as follows:

$$P_{Pt} = f(P_{Pt})$$

(23)

**Decomposition Sub-block**

We can break factors of CO$_2$ emissions as the following equation:

$$CO_{2,t} = \kappa_t \times \left( \frac{E_{It}}{E_{Pt}} \right) \times \left( \frac{E_{It}}{Z_t} \right) \times Z_t$$

(24)

where $\kappa_t = (CO_2/E_{It})$, $e_{It} = (E_{It}/E_{Pt})$ and $e_{Zt} = (E_{It}/Z_t)$. $\kappa_t$ is CO$_2$ emissions per one unit of the primary energy. High value of this indicator means a country consumes fossil fuels. $e_{It}$ shows the efficiency of energy
transformation. If it is high, the transformation is efficient. \( e_E \) is the energy intensity. If an economy has energy-saving structure, this should be low.

5. Preliminary Estimation Result by Using Genetic Algorithms

This section provides the preliminary estimation result of the US monetary policy (1991:1 – 1996:4) by using genetic algorithms. We consider that the US monetary policy instrument is the federal fund rate, and target variables are the inflation rate, the unemployment rate and the money growth rate. Since there are time lags for the effects of monetary policy, we assume that the Federal Reserve decides its monetary policy projecting the economic situations of three periods ahead. This US social welfare function \( F_{USA} \) can be written as:

\[
F_{USA} = w_1 \sum_{i=0}^{3} (GR4\_PCP92USA_{t+i} - GR4\_PCP92USA^*_{t+i})^2
+ w_2 \sum_{i=0}^{3} (URUSA_{t+i} - URUSA^*_{t+i})^2
+ w_3 \sum_{i=0}^{3} (GR\_M\_2USA_{t+i} - GR\_M\_2USA^*_{t+i})^2
+ w_4 (RFFUSA_t - RFFUSA^*)^2 ,
\]

where \( GR4\_PCP92USA_t \) is the four-period percentage change of the US private final consumption deflator (1992 = 100), \( URUSA_t \) is the US unemployment rate, \( GR\_M\_2USA_t \) is the percentage change of the US money supply, \( RFFUSA_t \) is the federal funds rate, \( w_1 \) is the weight of \( GR4\_PCP92USA_t \), \( w_2 \) is the weight of \( URUSA_t \), \( w_3 \) is the weight of \( GR\_M\_2USA_t \), \( w_4 \) is the weight of \( RFFUSA_t \), and variables with asterisks denote those desired values. In order to minimize equation (25) subject to the US macroeconometric model, we differentiate equation (25) with respect to \( RFFUSA \) and set the outcome equal 0 as follows:
Rearranging equation (26) yields the US monetary policy reaction function as:

\[
RFFUSA_t = RFFUSA_t^* \quad - \frac{w_1}{w_t} \sum_{i=0}^{3} \left( GR4 \_PCP92USA_{t+i} - GR4 \_PCP92USA_{t+i}^* \right) \frac{\partial GR4 \_PCP92USA_{t+i}}{\partial RFFUSA_t} \\
- \frac{w_2}{w_t} \sum_{i=0}^{3} \left( URUSA_{t+i} - URUSA_{t+i}^* \right) \frac{\partial URUSA_{t+i}}{\partial RFFUSA_t} \\
- \frac{w_3}{w_t} \sum_{i=0}^{3} \left( GR \_M 2USA_{t+i} - GR \_M 2USA_{t+i}^* \right) \frac{\partial GR \_M 2USA_{t+i}}{\partial RFFUSA_t}.
\] (27)

In order to estimate the US monetary policy reaction function, we rewrite equation (27) as:
Since each multiplier is pre-determined for an estimation, equation (28) can be written as:

\[
RFFUSA_t = \alpha_0 + \alpha_1 \sum_{i=0}^{3} \text{GR}_4 \_ \text{PCP92USA}_{t+i}^+ \sum_{i=0}^{3} \left( \text{GR}_4 \_ \text{PCP92USA}_{t+i}^+ \frac{\partial \text{GR}_4 \_ \text{PCP92USA}_{t+i}^+}{\partial RFFUSA_t} \right) \\
+ \frac{w_2}{w_f} \sum_{i=0}^{3} \left( \text{URUSA}_{t+i}^+ \frac{\partial \text{URUSA}_{t+i}^+}{\partial RFFUSA_t} \right) \\
+ \frac{w_3}{w_f} \sum_{i=0}^{3} \left( \text{GR}_4 \_ \text{M} \_2 \text{USA}_{t+i}^+ \frac{\partial \text{GR}_4 \_ \text{PCP92USA}_{t+i}^+}{\partial RFFUSA_t} \right) \\
- \frac{w_1}{w_f} \sum_{i=0}^{3} \left( \text{GR}_4 \_ \text{PCP92USA}_{t+i}^+ \frac{\partial \text{GR}_4 \_ \text{PCP92USA}_{t+i}^+}{\partial RFFUSA_t} \right) \\
- \frac{w_2}{w_f} \sum_{i=0}^{3} \left( \text{URUSA}_{t+i}^+ \frac{\partial \text{URUSA}_{t+i}^+}{\partial RFFUSA_t} \right) \\
- \frac{w_3}{w_f} \sum_{i=0}^{3} \left( \text{GR}_4 \_ \text{M} \_2 \text{USA}_{t+i}^+ \frac{\partial \text{GR}_4 \_ \text{M} \_2 \text{USA}_{t+i}^+}{\partial RFFUSA_t} \right).
\]

(28)

Since each multiplier is pre-determined for an estimation, equation (28) can be written as:

\[
RFFUSA_t = \alpha_0 + \alpha_1 \sum_{i=0}^{3} \text{GR}_4 \_ \text{PCP92USA}_{t+i} \\
+ \alpha_2 \sum_{i=0}^{3} \text{URUSA}_{t+i} + \alpha_3 \sum_{i=0}^{3} \text{GR}_4 \_ \text{M} \_2 \text{USA}_{t+i},
\]

(29)

\[
\alpha_0 = RFFUSA_t^+ + \frac{w_1}{w_f} \sum_{i=0}^{3} \frac{\partial \text{GR}_4 \_ \text{PCP92USA}_{t+i}^+}{\partial RFFUSA_t} \sum_{i=0}^{3} \text{GR}_4 \_ \text{PCP92USA}_{t+i}^+ \\
+ \frac{w_2}{w_f} \sum_{i=0}^{3} \frac{\partial \text{URUSA}_{t+i}^+}{\partial RFFUSA_t} \sum_{i=0}^{3} \text{URUSA}_{t+i}^+ \\
+ \frac{w_3}{w_f} \sum_{i=0}^{3} \frac{\partial \text{GR}_4 \_ \text{M} \_2 \text{USA}_{t+i}^+}{\partial RFFUSA_t} \sum_{i=0}^{3} \text{GR}_4 \_ \text{M} \_2 \text{USA}_{t+i}^+,
\]

where

\[
\alpha_1 = -\frac{w_1}{w_f} \sum_{i=0}^{3} \frac{\partial \text{GR}_4 \_ \text{PCP92USA}_{t+i}^+}{\partial RFFUSA_t}, \quad \alpha_2 = -\frac{w_2}{w_f} \sum_{i=0}^{3} \frac{\partial \text{URUSA}_{t+i}^+}{\partial RFFUSA_t}
\]

and

\[
\alpha_3 = -\frac{w_3}{w_f} \sum_{i=0}^{3} \frac{\partial \text{GR}_4 \_ \text{M} \_2 \text{USA}_{t+i}^+}{\partial RFFUSA_t}.
\]

Table 1 provides the estimation result of the US monetary policy reaction function (equation 29). P-values indicate that all variables are statistically significant at a 1 per cent level. Since the Durbin-Watson
statistic equals 0.773, it shows the positive first order autoregressive process. The estimation result shows \( \alpha_1 > 0, \alpha_2 < 0 \) and \( \alpha_3 > 0 \). The three multipliers must be \( \frac{\partial \text{GR4}_4 \cdot \text{PCP92USA}_{t+i}}{\partial \text{RFFUSA}_t} < 0 \), \( \frac{\partial \text{URUSA}_{t+i}}{\partial \text{RFFUSA}_t} > 0 \) and \( \frac{\partial \text{GR}_4 \cdot \text{M2USA}_{t+i}}{\partial \text{RFFUSA}_t} < 0 \) \((i = 0, 1, 2, 3)\). Hence, these estimates are theoretically consistent.

This policy reaction function can also explain the US monetary policy. Yet, there is a risk that a sudden and large policy change can lead to economic instability. Hence, we introduce a partial adjustment function in order to smooth movements of the federal funds rate. A partial adjustment mechanism can be written as:

\[
\text{RFFUSA}_t - \text{RFFUSA}_{t-1} = \lambda (\text{RFFUSA}_t^{**} - \text{RFFUSA}_{t-1}), \quad (30)
\]

where \( \lambda \) = adjustment parameter and \( \text{RFFUSA}_t^{**} \) = federal funds rate that is explained by the US monetary policy reaction function. Rearranging equation (30) yields

\[
\text{RFFUSA}_t = \lambda \cdot \text{RFFUSA}_t^{**} + (1 - \lambda) \text{RFFUSA}_{t-1}. \quad (31)
\]

This equation explains the federal funds rate.

Next, we explain the process on applying genetic algorithms for an analysis of the US monetary policy. This analysis uses 30 bits of 50 chromosomes. We assign the first to fifth bits for the constant term, sixth to tenth bits for \( \sum_{i=0}^{3} \text{GR4}_4 \cdot \text{PCP92USA}_{t+i} \), eleventh to fifteenth bits for \( \sum_{i=0}^{3} \text{URUSA}_{t+i} \), sixteenth to twentieth bits for \( \sum_{i=0}^{3} \text{GR}_4 \cdot \text{M2USA}_{t+i} \) and twenty-first to thirtieth bits for \( \lambda \). Weights of the fitness value function vary between the plus and minus standard error of each parameter obtained by regression analysis. The fitness value function of the US monetary policy can be written as follows:
\[ VALUSA_{j,t}^n = VALUSA_{0,j,t} + VALUSA_{1,j,t} + VALUSA_{2,j,t} + VALUSA_{3,j,t} \]  

(32)

where

\[ VALUSA_{0,j,t} = 3.721148 + chromusa_{1,j,t} \cdot -0.73 + chromusa_{2,j,t} \cdot -0.365 + chromusa_{3,j,t} \cdot 0 \]

\[ + chromusa_{4,j,t} \cdot 0.365 + chromusa_{5,j,t} \cdot 0.73 \]  

(33)

\[ VALUSA_{1,j,t} = 30.04511 \cdot \sum_{i=0}^{3} GR4\_PCP\_USA_{i+j} \]

\[ + chromusa_{6,j,t} \cdot \sum_{i=0}^{3} GR4\_PCP\_USA_{i+j} \cdot -1.7 \]

\[ + chromusa_{7,j,t} \cdot \sum_{i=0}^{3} GR4\_PCP\_USA_{i+j} \cdot -0.85 \]

\[ + chromusa_{8,j,t} \cdot \sum_{i=0}^{3} GR4\_PCP\_USA_{i+j} \cdot 0 \]

\[ + chromusa_{9,j,t} \cdot \sum_{i=0}^{3} GR4\_PCP\_USA_{i+j} \cdot 0.85 \]

\[ + chromusa_{10,j,t} \cdot \sum_{i=0}^{3} GR4\_PCP\_USA_{i+j} \cdot 1.7 \]  

(34)

\[ VALUSA_{2,j,t} = -0.134334 \sum_{i=0}^{3} URUSA_{i+j} + chromusa_{11,j,t} \sum_{i=0}^{3} URUSA_{i+j} \cdot -0.02 \]

\[ + chromusa_{12,j,t} \sum_{i=0}^{3} URUSA_{i+j} \cdot 0.01 + chromusa_{13,j,t} \sum_{i=0}^{3} URUSA_{i+j} \cdot 0 \]

\[ + chromusa_{14,j,t} \sum_{i=0}^{3} URUSA_{i+j} \cdot 0.01 + chromusa_{15,j,t} \sum_{i=0}^{3} URUSA_{i+j} \cdot 0.02 \]  

(35)
\[ VALUSA_{3,j,t} = 41.64614 \sum_{i=0}^{3} GR_{-M2USA_i} + chromusa_{16,j,t} \sum_{i=0}^{3} GR_{-MUSA_i} \cdot -3.7 \\
+ chromusa_{17,j,t} \sum_{i=0}^{3} GR_{-M2USA_i} \cdot -1.85 \\
+ chromusa_{18,j,t} \sum_{i=0}^{3} GR_{-M2USA_i} \cdot 0 \\
+ chromusa_{19,j,t} \sum_{i=0}^{3} GR_{-M2USA_i} \cdot 1.85 \\
+ chromusa_{20,j,t} \sum_{i=0}^{3} GR_{-M2USA_i} \cdot 3.7 \]  

(36)

and \( chromusa_{nbit, j, t} \) denotes the \( n \)th bit of the \( j \)th individual of the US chromosome at time \( t \).

Regarding an adjustment parameter, all weights are 0.1. Thus, when values of the ten bits are 1, it becomes 1. The partial adjustment function can be written as:

\[ RFFUSA_{j,t} = \lambda_{j,t} \cdot VALUSA_{j,t} + (1 - \lambda_{j,t}) RFFUSA_{j,t-1}, \]  

(37)

where \( \lambda_{j,t} = \sum_{nbit=21}^{30} chromusa_{nbit, j, t} \cdot 0.1 \).

As shown in equation (37), the federal funds rate is estimated for each individual of a chromosome and time. Hence, we evaluate each computed value of the federal funds rate by the following function:

\[ VALUSA_{-F, j, t} = \frac{1}{(RFFUSA_{j,t} - RFFUSA_{j,t-1})^2}. \]  

(38)

We select the chromosome that maximizes equation (38) as the best chromosome and compute the
optimal federal funds rate by using this maximized chromosome. Table 2 shows the parameters computed by genetic algorithms. Figure 1 provides estimated federal funds rate by genetic algorithms. We note that the crossover rate and the mutation rate are set as 0.25 and 0.01, respectively, and the number of the iteration is five.

We apply this methodology in order to assess the optimal \( \text{CO}_2 \) abatement policies in favor of the Kyoto Protocol.

Notes:

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\( ^b \) E-mail: hkosaka@sfc.keio.ac.jp

1 According to our calculation based on EDMC (2001), these eight countries released roughly 40 percent of \( \text{CO}_2 \) worldwide in 1990.

2 This paper focuses on \( \text{CO}_2 \) emissions among GHGs because approximately 60 percent of GHGs is carbon dioxide emissions.

3 As for other explanations on genetic algorithms, see Goldberg (1989) and Michalewicz (1996).

4 As for examples, see Fair (1984, 1994).

5 This energy model is provided in Kosaka (1994).

6 Formulation of this two-level CES production function is based on Lu and Kaya (1989).

References


Nordhaus, William D., 1994, Managing the Global Commons: The Economics of Climate Change,
Cambridge, Mass.: MIT Press.


Table 1  Estimation Result of the US Monetary Policy Reaction Function

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>t-statistic</th>
<th>p-value</th>
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</thead>
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<tr>
<td>α₀</td>
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<td>5.100</td>
<td>0.000</td>
</tr>
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<td>α₁</td>
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<tr>
<td>α₂</td>
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</tr>
<tr>
<td>α₃</td>
<td>41.646</td>
<td>11.315</td>
<td>0.000</td>
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</table>

Adj. $R^2$ 0.923
S.E. 0.517
D.W. 0.773

Estimation technique: Ordinary least squares

Table 2  Parameters Computed by Genetic Algorithms

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<tr>
<th>Year</th>
<th>α₀</th>
<th>α₁</th>
<th>α₂</th>
<th>α₃</th>
<th>λ</th>
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<td>-0.144</td>
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</tr>
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<tr>
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<tr>
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<tr>
<td>Year</td>
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<td>Rate 2</td>
<td>Rate 3</td>
<td>Rate 4</td>
<td>Rate 5</td>
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<td>------</td>
<td>--------</td>
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Note: RFFUSA_GA = the federal funds rate that is computed by genetic algorithms.