Abstract

The standard estimator introduced by Blanchflower and Oswald (1994) to determine the short run unemployment elasticity of pay controls for both regional and time period fixed effects. This paper identifies two general cases in which the estimator that controls for regional fixed effects but not for time period fixed effects offers a better estimate of this elasticity: (1) the correlation coefficients of the unemployment rate observed at single regions over time are large and diminish slightly over time, and (2) the national unemployment rate is high.

The standard estimator also takes no account of the spatial relationship among regions. Ignoring this relationship may seriously bias the results. To investigate this, the East German wage curve is estimated including spatial effects using panel data classified into 114 administrative districts over the period 1993-1999. Moreover, we also control for the possibility that the unemployment rate is correlated with the disturbance term.

The short run unemployment elasticity of pay found for Eastern Germany amounts to -0.112, a figure very close to the -0.10 posited by Blanchflower and Oswald, though its foundation is completely different. In contrast to Blanchflower and Oswald, our figure is corrected for spatial effects and also captures the downward effect of the national unemployment rate. Without them, the unemployment elasticity would successively more than double at -0.242 or shrink to -0.006.

KEYWORDS: Wage Curve, Panel Data, Spatial Autocorrelation, Instrumental Variables, Short Run, Long Run, Eastern Germany.

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1. INTRODUCTION

This paper presents new evidence on the wage curve. The wage curve describes the individual wage rate as a downward-sloping convex curve of the regional unemployment rate. It is an empirical relationship established by Blanchflower and Oswald (1994) for a couple of countries and since then by many others.\(^1\) The existence of a wage curve implies that people who work in labor markets with higher unemployment earn a substantially lower wage. The wage curve seems to disprove the conventional wisdom that high wages compensate workers for high unemployment across regions that has dominated much economic thinking. We deliberately used the verb “seem”, as Blanchflower and Oswald (1994, p.181) consider the wage curve as a short run concept and the compensating differential as a long run concept.

We have three reasons to reexamine the wage curve. Blanchflower and Oswald have paid much attention to the question whether or not regional fixed effects should be included in the regression equation. The first reason to reexamine the wage curve is that the question whether or not to include time dummies is as important as the question whether or not to include regional fixed effects. A common conjecture of the older applied econometrics literature is that time series data tend to yield short run responses while cross section data tend to yield long run responses. In addition, the panel data literature has pointed out that the estimator that only utilizes the variation within the observations over time, but not between the observations, offers a better estimate of short run effects, while the estimator that only utilizes the variation between the observations, but not within the observations over time, offers a better estimate of long run effects (Baltagi and Griffin, 1984). This seems to imply that the estimator that controls for regional fixed effects, but not for time dummies, is the right one to test the short run concept of the wage curve and that the estimator that controls for time dummies, but not for regional fixed effects, is the right one to test the long run concept of the compensating differential.

The second reason to reexamine the wage curve is that time dummies have the side effect to eliminate the national unemployment effect. Regressing the wage rate \(w\) on the regional unemployment rate \(u\) and a set of time dummies, has the effect that variation within the observations over time will no longer be utilized in forming the estimates of the response parameters. Provided that the regression covers all regions within the country under study and that each region is weighted by its number of workers, it is computational equivalent with regressing \(w\) on \(u\), both in deviation from their national counterparts. Consequently, the model does not capture the potential effect of national unemployment, while it might be expected that the hypothesized downward effect of unemployment in a downswing of the national economy is greater than in an upswing of the national economy.

\(^1\) For a review on some of the benefits and limitations of the wage curve we mention Card (1995). For more German studies we mention Wagner (1994), Baltagi and Blien (1998), Baltagi et al. (2000), and Pannenberg and Schwarze (1998).
The third reason to reexamine the wage curve is that, up to now, nobody has taken account of the spatial relationship among regions. Recently, Bell and Bocksteal (2000) have pointed out that ignoring the spatial relationship among observations is analogous to ignoring the ordering of time series data. The first element being ignored is that distance affects household behavior. Regional science theory has pointed out that households may change their location, consumption and labor supply decisions, depending on the market conditions in the home region compared to other regions and on the distance to these regions. The regional unemployment rate used to explain the wage rate is the unemployment rate observed in the employee’s home region (Blanchflower and Oswald, 1994, p.5). However, an employee may work and live in two different regions. If the wage rate in a nearby region is higher due to lower unemployment and this higher wage rate compensates for the time and travel costs of commuting, an employee may supply its labor outside his home region. This means that the unemployment rate measured in the home region may not be the relevant unemployment rate that captures the effect on wage determination.

The second element being ignored is that data collection of observations associated with spatial units might reflect measurement error. This would occur if the administrative boundaries for collecting information— the arbitrary delineation of space into different units (countries, states, provinces, counties, tracts or zip codes)—do not accurately reflect the nature of the underlying process generating the sample data. We already pointed out that the unemployment rate measured in the home region might not be the relevant one if large numbers of workers travel up and down daily from one region to another. This happens when information is collected from relatively small regions. On the other hand, when information is collected from relatively large regions, the regional unemployment rate may also deviate from the relevant unemployment rate, due to unobserved intraregional differences.\footnote{This problem disappears when information is collected from travel-to-work areas, but in practice most studies depart from administratively defined areas, often in order to fulfil another and maybe even more important requirement, i.e., to have access to enough data. In this respect, Blanchflower and Oswald’s work is no exception.}

This paper consists of three parts: a theoretical, a methodological and an empirical part. The theoretical part briefly discusses the theory behind and the current state of research on the wage curve and explains how to determine the short and long run unemployment elasticity. The methodological part introduces a couple of panel data models that are frequently used in applied research on the wage curve and a framework to determine the most likely candidate, including panel data models geared to instrumental variable methods. As the regional unemployment rate might be correlated with the disturbance term, instrumental variable methods like two-stage least squares (2SLS) are required to obtain consistent parameter estimates. In contrast to Blanchflower and Oswald (1994), but in accordance with Baltagi et al. (2000), we find that instrumental variable methods have a significant effect on the empirical results. In addition to Baltagi et al. (2000), we not only test for exogeneity of the regressors and whether or not the regional fixed effects are correlated with the regressors,
but also whether or not the time dummies are correlated with the regressors. Next, the panel
data models are extended by the possibility of spatially correlated error terms. Instead of the
commonly used first-order spatial autocorrelation model, we adopt the use of matrix
exponential for spatially transforming the error term, a transformation recently introduced by
Pace and LeSage (2001). It is shown that this transformation has many advantages. In the
empirical part the East German wage curve is estimated using data classified into 114
administrative districts over the period 1993-1999. To describe the spatial relationship among
the districts, a 114×114 matrix is constructed based on commuting flow data.

2. THE WAGE CURVE: SHORT RUN VERSUS LONG RUN EFFECTS

Although different functional forms of the wage curve have been investigated, the double log
form dominates the literature

\[
\log w_{irt} = \alpha_0 + \alpha_1 \log u_{rt} + Z_{irt} \gamma + \varepsilon_{irt},
\]

where \( w_{irt} \) is the wage rate of individual \( i \) observed in region \( r \) at period \( t \) \((r=1,…,N; t=1,…,T)\). \( u_{rt} \) is the unemployment rate in region \( r \) at time period \( t \). \( Z_{irt} \) is a set of additional
explanatory variables of a kind conventional in the literature on cross-section wage equations.
\( \varepsilon_{irt} \) is the error term.

It is important to note that \( u_{rt} \) does not vary with \( i \), which implies that the relevant
dimension for the estimation of the unemployment coefficient is not the number of individual
wage observations times the number of time periods, but the number of regional labor
markets times the number of time periods. Also it implies that \( \varepsilon_{irt} \) will be correlated across
observations within the same region, as a result of which the standard error of the
unemployment coefficient will be biased downward (Moulton, 1990). One remedy is to
average over individuals in region \( r \) at period \( t \) to get

\[
\log w_{rt} = \alpha_0 + \alpha_1 \log u_{rt} + Z_{rt} \gamma + \varepsilon_{rt} = X_{rt} \beta + \varepsilon_{rt},
\]

where \( \log w_{rt} \) represents the average log wage for all individuals in region \( r \) at time period \( t \)
and \( Z_{rt} \) and \( \varepsilon_{rt} \) are similar averages over all individuals in region \( r \) at time period \( t \). Equation
(2) with \( X_{rt} = (1, \log u_{rt}, Z_{rt}) \) can be estimated using region-by-year “cell means”, one of the
approaches which was followed by Blanchflower and Oswald (1994, pp.167-177) and which
will also be followed in this paper. Assuming that there is no correlation in the unobserved
determinants of wages across regions — an assumption which is rather restrictive and subject
to discussion in the section on spatial effects —, the residuals in equation (2) are uncorrelated
across observations, and conventional standard error formulas are valid.
The regression model that dominates the wage curve literature is the “effects” model

\[
\log w_{rt} = \mu_r + \lambda_t + X_{rt}\beta + \epsilon_{rt},
\]

in which variation across regions \((\mu_r)\) or time \((\lambda_t)\) is captured in changes of the intercept. The reasoning behind these effects has been spelled out in the panel data literature. Different regions and different time periods might have different backgrounds, i.e., region-specific time-invariant and time-specific region-invariant variables that do affect the dependent variable of the analysis but are difficult to measure or hard to obtain. To fail to account for these variables runs the risk of obtaining biased results. One remedy is to control for regional and time period effects.

It is less well-known that the panel data literature has concurrently pointed out that controlling for regional and time period effects, dependent on the phenomena being modeled and the nature of the data set, may not always have the desired effect (Kuh, 1959; Houthakker, 1965; Baltagi and Griffin, 1984; Mairesse, 1990; Baltagi, 2001, pp.197-198). Applied researchers are often puzzled with the significant difference among the coefficient estimates — in wage curve regressions among the unemployment elasticity estimate — from the Within, the Between and the OLS estimators. These estimators are different in that they utilize different parts of the variation between the observations. Applied studies controlling for regional effects \((\mu_r)\) find that the Within estimator (which is based on the time-series component of the data) tends to give short-run estimates, whereas the Between estimator (which is based on the cross-sectional component of the data) tends to give long-run estimates. Conversely, applied studies controlling for time period effects \((\lambda_t)\) find that the Within estimator (which is based on the cross-sectional component of the data) tends to give long-run estimates, whereas the Between estimator (which is based on the time-series component of the data) tends to give short-run estimates. In both cases the OLS estimator can be considered a compromise between the Within and the Between estimator as it is based on the time-series as well as the cross-sectional component of the data.

Baltagi and Griffin (1984) have pointed out that the difference among the coefficient estimates from the Within and Between estimators can be explained by dynamic underspecification. The basic idea is that even with a rich panel data set, long-lived lag effects coupled with limited time series observations is a recipe to dynamic underspecification.\(^3\) Although Blanchflower and Oswald have investigated different functional forms, none of the micro-economic regressions in their book contains lagged dependent and independent variables, simply because these variables were not available at this level. Only in the region-by-year cell means regressions lag effects up to a second degree have been taken up. Comparison of the correlation coefficients between \(X_{it}, X_{it-1}, \ldots, X_{it}\) offers a practical guide to the extent of underspecification. The higher (smaller) the correlation, the

\(^3\) This is illustrated using Monte Carlo experiments on a double log functional form.
higher (smaller) the probability of dynamic underspecification. The question is what to do if dynamic underspecification is likely to be present.

Baltagi and Griffin (1984) have pointed out that one should first compare the variation between the observations at one point in time with the variation within the observations over time for the relevant explanatory variables. If the former exceeds the latter, the preferred estimator of short-run effects can be obtained by controlling for regional fixed effects (Within estimator), but not for time period fixed effects (Between or OLS estimator), so as to fully draw on the time-series component of the data. By contrast, the preferred estimator of long-run effects can be obtained by controlling for time period fixed effects (Within estimator), but not for regional fixed effects (Between of OLS estimator), so as to fully draw on the cross-sectional component of the data. The choice whether to use the Between or the OLS estimator depends on the extent to which the variation between the observations at one point in time exceeds the variation between the observations over time. The greater this ratio, the better the Between estimator. We will use these findings to investigate the concepts of the wage curve and the unemployment differential, respectively by estimating the short-run and the long-run unemployment elasticity of pay.

3. PANEL DATA MODELS

A difficult choice in empirical research on the wage curve is the type of panel data model and its corresponding estimator. Figure 1 presents a framework to determine the best of a series of candidates. This figure has been constructed using Baltagi (2001) and Keane and Runkle (1992). Starting point in this figure is the model without fixed or random regional and time period effects. This model can be estimated using OLS.

The objection to this model is that it does not control for persistent unobserved differences among regions, which runs the risk of obtaining biased results. One remedy is to transform the model into a fixed effects model — the dominant approach in wage curve regressions — or into a random effects model. In the fixed effects model (FE), a dummy variable is introduced for each region, while in the random effects model (RE), the intercept is treated as a random variable. To determine the right estimator, the FE estimator should be tested against OLS, the RE estimator against OLS, and the RE estimator against the FE estimator. However, one should be careful to endorse one of these three estimators, as one or more of the right-hand side regressors might be correlated with the disturbance term. This may be due to the omission of relevant variables, measurement error, sample selectivity, self-selection or other reasons. Endogeneity causes inconsistency of the OLS, FE and RE estimates and requires instrumental variable methods like two-stage least squares (2SLS) to obtain consistent parameter estimates. To control for the possible endogeneity of certain explanatory variables, the 2SLS, the fixed effects 2SLS (FE-2SLS) and the first-differenced 2SLS (FD-2SLS) estimators should be run, using as instruments the exogenous variables and the lagged values of the exogenous
variables as well as of the endogenous variable. To determine the right estimator, Keane and Runkle (1992) have suggested testing whether the explanatory variables are merely predetermined or strictly exogenous and testing whether or not the fixed effects are correlated with the explanatory variables.

Keane and Runkle’s specification test for strict exogeneity of the explanatory variables (the null hypothesis) is $\hat{\beta}_{\text{FE2SLS}} - \hat{\beta}_{\text{FD2SLS}}$\bigg($\hat{\beta}_{\text{FE2SLS}} - \hat{\beta}_{\text{FD2SLS}}\bigg)^{-1}\hat{\beta}_{\text{FE2SLS}} - \hat{\beta}_{\text{FD2SLS}}$, which is distributed asymptotically as a $\chi^2_k$ random variable if $\hat{\beta}_{\text{FE2SLS}}$ and $\hat{\beta}_{\text{FD2SLS}}$ each contain $k$ parameters. Consider the first-difference equation $Y_{it} = (X_{it} - X_{it-1})\beta + \epsilon_{it} - \epsilon_{it-1}$ (i=1,…,N; t=2,…,T). Keane and Runkle have pointed out that the FD-2SLS estimator will give a consistent estimate of $\beta$ whether or not the explanatory variables are strictly exogenous. Furthermore, as first differencing eliminates any potential unobserved regional fixed effect, there will also be no problem caused by correlation of a regional fixed effect with the explanatory variables. By contrast, the FE-2SLS estimator will give a consistent estimate of $\beta$ only if the explanatory variables are strictly exogenous.

In case the explanatory variables appear to be strictly exogenous, instrumental variable methods are not needed and the choice between the OLS, FE and RE estimator suffices. In case (part of) the explanatory variables appear to be predetermined instead of strictly exogenous, Keane and Runkle’s specification test for regional fixed effects — the null hypothesis is that no regional fixed effect is correlated with the explanatory variables — is $\hat{\beta}_{2SLS} - \hat{\beta}_{\text{FD2SLS}}$\bigg($\hat{\beta}_{2SLS} - \hat{\beta}_{\text{FD2SLS}}\bigg)^{-1}(\hat{\beta}_{2SLS} - \hat{\beta}_{\text{FD2SLS}})$, which is distributed asymptotically as a $\chi^2_k$ random variable if $\hat{\beta}_{2SLS}$ and $\hat{\beta}_{\text{FD2SLS}}$ each contain $k$ parameters. Again the FD-2SLS estimator will give a consistent estimate of $\beta$ whether or not the null hypothesis is rejected. By contrast, the 2SLS estimator will give a consistent estimate of $\beta$ only if the null hypothesis is not rejected.

Using this framework, Baltagi et al. (2000) conclude that FD-2SLS is the preferred estimator of the Eastern German wage curve, since the regional unemployment rate variable does not appear to be strictly exogenous and the regional fixed effects do not appear to be uncorrelated with the explanatory variables. Remarkably, a similar framework to find out how to deal with potential region-invariant unobserved differences among time periods has not been used. Time period fixed effects are, in fact, added to the regression equation without comment. The same applies to Blanchflower and Oswald (1994). They present several wage curve regressions with and without regional fixed effects, indicating that the inclusion of regional fixed effects is subject to discussion, but not any wage curve regressions without time period fixed effects. This is striking as the resulting estimator due to the inclusion of time period effects draws on the cross-sectional component of the data, whereas the wage curve is considered a short-run concept. In this paper we will remove this defect; First, by also testing how to deal with potential region-invariant unobserved differences among time

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4 The kinds of tests that can be used are spelled out in Baltagi (2001).
5 The 2SLS estimator is standard; the FE-2SLS and FD-2SLS estimators are spelled out in Baltagi (2001, ch.7 and ch.8).
periods and, second, by better matching the inclusion of regional or time period fixed effects with the desired short-run or long-run estimate of the unemployment elasticity.

To test for potential region-invariant unobserved differences among time periods, a similar framework is used as in figure 1, except for the FD-2SLS estimator. First-differencing eliminates regional fixed effects, the purpose of this transformation, but this transformation does not help to eliminate time period fixed effects. As an alternative, we introduce the spatially first-differenced 2SLS (SFD-2SLS) estimator based on the regression equation

$$\log w_{it} - \log w_{0t} = (X_{it} - X_{0t})' \beta + \varepsilon_{it} - \varepsilon_{0t},$$  

(4)

where the values of Y and X in every region are taken in deviation of Y and X in one reference region 0. Just as first differencing in time diminishes the number of observations available for estimation, so does first differencing in space; the former by one for every region and the latter by one for every time period. Another difference touches the covariance matrix. The covariance matrix used to determine the FD-2SLS estimator is (Baltagi, 2001, p.132)

$$E(\Delta \varepsilon \Delta \varepsilon') = \sigma^2 (I_N \otimes G),$$

where G is a (T-1)×(T-1) matrix, $G = \begin{pmatrix}
2 & -1 & 0 & 0 \\
-1 & 2 & -1 & 0 \\
0 & -1 & 2 & 0 \\
0 & 0 & 0 & 2
\end{pmatrix}$

(5)

The covariance matrix we have derived for the SFD-2SLS estimator is

$$E((\varepsilon - \varepsilon_0)(\varepsilon - \varepsilon_0)') = \sigma^2 (H \otimes I_N),$$.  

where H is a (N-1)×(N-1) matrix, $H = I_{N-1} + e_{N-1}e_{N-1}'$,  

(6)

and $e_{N-1}$ is a (N-1)×1 vector of ones. The inverse of H is

$$H^{-1} = I_{N-1} - \frac{1}{N} e_{N-1}e_{N-1}'$$

(7)

This implies that the SFD-2SLS estimator of $\beta$ in (4) is equivalent to the 2SLS estimator of the regressions equation $\log w_{it} = X_{it}^0 + \varepsilon_{it}$, with $\varepsilon_{it} \sim N(0, \sigma^2)$, 

$$\log w_{it}^* = \log w_{it} + \left(\frac{1}{\sqrt{N}} - 1\right) \log \bar{w}_i, \quad \text{and} \quad X_{it}^* = X_{it} + \left(\frac{1}{\sqrt{N}} - 1\right) \overline{X}_i.$$  

(8)
4. SPATIAL EFFECTS

Equation (2) can be estimated using region-by-year cell means assuming that there is no correlation among the unobserved determinants of wages across regions. However, this assumption is not very likely. Not only people from the same region, but also people from two different regions may share the same unobservable characteristics. In many countries labor market institutions, such as the wage bargaining, social security, retirement and tax systems, do not differ to any great extent between regions. All of these country-specific circumstances affect the level around which the regional wage rates within one country vary. Wage curve regressions covering only one country are not able to measure the effects of these circumstances, which also explains why they usually are not observed. Nevertheless, it would be wrong to assume that they are not there. In addition to this, variation in the wage distribution among different parts of the country might be explained by unobserved sub-national variables, such as differences in unionization, differences in the accessibility of regions which in turn affect the possibility of workers to supply their labor outside their home region, and the fact that some parts of the country may be separate political entities.

National and sub-national unobserved circumstances might have different regional effects, as a result of which regional error terms correlate and their variance is not constant across the country. The extent to which these national and sub-national circumstances cause the regional error terms to affect each other depends on the spatial arrangement of the regions, usually embodied by a spatial weight matrix. Most of the theoretical results in spatial econometrics relate to spatial weight matrices whose elements are posited as being 1 if two regions share a common border and 0 otherwise, or as being inverse functions of the distance between regions. In this study, the elements of the spatial weight matrix are based on commuting flow data, \( W = \{w_{ij}\} \), where \( w_{ij} \) represents the number of people living in region \( i \) and working in region \( j \). In we further take \( w_{ij} \) in relation to all people working outside their home region, each row sum equals one, known as row-normalizing. The diagonal elements of \( W \) are set to zero, as a regional error term cannot be spatially correlated with itself.

The best-known model focusing on spatial dependence starts with a first-order spatial autoregressive process generating the error terms \( \varepsilon_t = \delta W \varepsilon_t + u_t \), where \( u_t \sim N(0, \sigma^2 I_N) \), the error terms \( \varepsilon_t \) and \( u_t \) are written in vector-form for each cross-section of regions at one point in time, and \( \delta \) is called the spatial autocorrelation coefficient (see Anselin, 1988, ch.8). Instead of the first-order spatial autocorrelation model, we adopt the use of matrix exponentials for spatially transforming the error term, a transformation recently introduced by Pace and LeSage (2001) and abbreviated to MESS (Matrix Exponential Spatial Specification). On using matrix exponentials, the error term \( \varepsilon_t \) undergoes a linear transformation \( S \varepsilon_t = u_t \) with \( S \) defined as (Chiu et al., 1996)
This means that
\[
\varepsilon_i = \delta W\varepsilon_i - \frac{\delta^2}{2} W^2\varepsilon_i + \frac{\delta^3}{6} W^3\varepsilon_i - \frac{\delta^4}{24} W^4\varepsilon_i + \frac{\delta^5}{120} W^5\varepsilon_i - \frac{\delta^6}{720} W^6\varepsilon_i + \frac{\delta^7}{5040} W^7\varepsilon_i + S(q \geq 8) + u_i. \tag{10}
\]

where \(S(q \geq 8)\) is considered a rest term. Note that MESS reduces to the first-order spatial autocorrelation model if the power series expansion is truncated after the first term. The advantages of MESS are many. The first advantage is that MESS not only takes account of first-order, but also of higher-order spatial effects. The second advantage is that the Jacobian term of transforming the estimation model from the error term into the dependent variable equals zero

\[
\log |S| = \log |e^{-\delta W}| = \log |e^{-\delta \text{trace}(W)}| = \log |e^{-\delta 0}| = \log |1| = 0. \tag{11}
\]

By contrast, the Jacobian term of the first-order autocorrelation model is

\[
T \log |I_N - \delta W| = \sum_{i=1}^{N} (1 - \delta \omega_i), \tag{12}
\]

where \(\omega_i\) \((i=1,\ldots,N)\) represents the eigenvalues of the spatial weight matrix \(W\). The numerical procedures required to compute these eigenvalues decreases rapidly as the size of \(W\) increases.\(^6\) Furthermore, the first-order autocorrelation model requires the spatial weight matrix before row-normalizing to be symmetric, otherwise the eigenvalues are not real but imaginary.\(^7\) On using MESS, both problems disappear. MESS does not require the computation of the eigenvalues and does not require the spatial weight matrix before row-normalizing to be symmetric. This implies that MESS can also handle an asymmetric spatial weight matrix based on commuting flow data.

A full description of MESS and its justification can be found in Pace and LeSage (2001). They have also published a MATLAB function on www.spatial-econometrics.com to run the model \(SY_i = X\beta + \varepsilon_i\) and \(\varepsilon_i \sim (0, \sigma^2 I_N)\). We have rewritten this function to run the model \(\log w_i = X\beta + \varepsilon_i\) with spatial effects \(s\varepsilon_i = u_i\) and \(u_i \sim (0, \sigma^2 I_N)\). To estimate the parameters, an iterative two-step procedure is used in which \(\beta\) (together with \(\sigma^2\)) and \(\delta\) are alternately estimated until convergence occurs. \(\beta\) and \(\sigma^2\), given \(\delta\), are estimated using OLS on the transformed data \(Y^* = SY\) and \(X^* = SX\). Note that this transformation is only practicable if the power series expansion of \(S\) is truncated after \(q\) terms. Following Pace and LeSage

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\(^6\) Kelejian and Prucha (1999) have pointed out that this might be problematic even for moderate sample sizes (\(N=400\)).

\(^7\) If a symmetric matrix is row-normalized its eigenvalues remain real (see Ord, 1975).
(2001), it has been decided to drop S(q≥8), as the impact of eight or higher order terms appeared to be negligible. \( \delta \), given \( \beta \) and \( \sigma^2 \), can be solved from its first-order maximizing log-likelihood condition, which is relatively easy in that a closed form solution for \( \delta \) exists.\(^8\)

The final advantage of MESS is that similar procedures can be used to determine the FE, 2SLS, FE-2SLS and FD-2SLS estimators. The standard method of estimating the fixed effects (FE) model is to eliminate the intercept and the fixed effects from the regression equation by demeaning the left-hand side variable as well as the right-hand side variables, and then to estimate the demeaned equation by OLS (Baltagi, 2001, pp.12-15). This demeaned equation can simply be extended to MESS along the lines described above. In case of 2SLS, we have a linear model \( Y = X\beta + \varepsilon \), with \( E(\varepsilon\varepsilon') = \sigma^2\Omega \), and one or more endogenous \( X \) variables. Let \( P \) denote the matrix of instrumental variables. Then the GLS analog instrumental variables estimator is (Bowden and Turkington, 1984, ch.3; Amemiya, 1985, pp.240-241)

\[
\hat{\beta} = (\hat{X}'\Omega^{-1}\hat{X})^{-1}\hat{X}'\Omega^{-1}Y, \quad \text{with} \quad \hat{X} = P(P'\Omega^{-1}P)^{-1}P\Omega^{-1}X, \tag{13}
\]

which can be seen as the result of a double application of generalized least squares. The problem is that this estimator appears to require an estimate of \( \Omega \), which depends on \( \delta \), \( \Omega = \Omega(\delta) \), is in hand already, whereas \( \delta \) must be estimated too. However, \( \delta \) can be estimated along the same line as in the linear regression model with spatial effects, since the Jacobian term in the latter model equals zero. The FE-2SLS estimator can be seen as a straightforward combination of the FE and the 2SLS estimators. The FD-2SLS estimator is partly equivalent to the 2SLS estimator, since the fixed effects have been eliminated from the regression equation by first-differencing. The difference is that the covariance matrix not only needs to be adjusted for MESS, but also for the \( G \)-matrix in (5).

Unfortunately, similar procedures cannot be used in case of the RE and SFD2SLS estimators, since the Jacobian term of these estimators no longer equals zero\(^9\), but in the empirical application below these estimators appear to be of minor importance.

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\(^8\) This also contrasts the first-order autocorrelation model in which \( \delta \) must be estimated by numerical optimization of the concentrated log likelihood function of \( \delta \), as \( \delta \) cannot be solved analytically from its first-order maximizing condition (Anselin, 1988: 181-182; Anselin and Hudak, 1992).

\(^9\) Anselin (1988, pp.150-156) and Baltagi (2001, pp.195-197) show that the Jacobian term in the first-order autocorrelation random effects models is

\[
T \log |I_N - \delta W| - \frac{1}{2} \sum_{i=1}^{N} \log(1 + T\theta^2 (1 - \delta \omega_i)),
\]

where \( \theta \) is an additional parameter to be estimated measuring the ratio of the variance of the regional or the time period random effects and the variance of the remainder error term \( u \). On using MESS, the first term disappears, just as in equation (11), while the second term changes into
5. THE EAST GERMAN WAGE CURVE

5.1 IMPLEMENTATION

For this study the East German wage curve is estimated over the period 1993-1999 including spatial effects. A description of the database can be found in Baltagi et al. (2000), who used the same database to estimate the East German wage curve over the period 1993-1998 but then without spatial effects. Nevertheless, there are some notable differences. First, it has been decided to use an updated regional delineation. In 1999/2000 the 114 administrative districts (“Landkreise & kreisfreie Städte”) were partly redefined. In contrast to Baltagi et al. (2000), our data set reflects the current situation, including the period 1993-1998. Due to this redefinition, unemployment rates had to be recalculated as well. This was done very carefully in a separate project in which we also found some data errors (Blien and Wolf, 2002). Second, we slightly changed the number of control variables: age, age squared, 1 gender dummy, 7 employment status categories, 5 (instead of 6) worker’s qualification categories, 27 (instead of 33) industry dummies, 12 (instead of 13) occupational dummies, and 6 establishment size categories.\(^{10}\) In addition, we have constructed a spatial weight matrix based on commuting flow data to test for spatially correlated error terms.

5.2 RESULTS

Table 1 reports the estimation results when time period fixed effects are included and regional fixed effects are tested for. This is the approach followed by Blanchflower and Oswald (1994), later extended by Baltagi et al. (2000). Using the test structure denoted in figure 1, we find that the estimator that controls for regional fixed effects and time period fixed effects, the latter by assuming that time period fixed effects should be included, is the preferred estimator. The hypothesis that the unemployment rate is strictly exogenous cannot be rejected (based on the first Hausman test), as a result of which instrumental variable methods are not required. The second Hausman test rejects the RE model in favor of the FE model, indicating that the regional fixed effects are correlated with the explanatory variables.\(^{11}\) In addition to this, we find that the FE estimator cannot be rejected in favor of the

\[
-\frac{1}{2} \sum_{i=1}^{N} \log(T0^2 + e^{-2\delta_n}).
\]

As this term does not disappear, it complicates a straightforward analysis of instrumental variables. The SFD-2SLS is mathematically even more problematic.

\(^{10}\) The number of control variables corresponding to each set of dummy or category variables has already been diminished by one.

\(^{11}\) As the RE estimator corrected for spatial effects is not available, we implemented the Hausman test based on the difference between the FD-2SLS and the 2SLS estimators in the right column of table 1 (see also figure 1).
FE estimator corrected for spatial effects. All of these results confirm the approach followed by Blanchflower and Oswald (1994). The only, though fundamental, difference is that they have found the unemployment elasticity of pay to be approximately -0.10, whereas we find a figure of only -0.006. As our figure also does not appear to be statistically different from zero, it seems that we have not found evidence in favor of the wage curve.

The question, however, is whether this figure really approaches the short-run unemployment elasticity of pay. One problem is that the inclusion of time period fixed effects has eliminated the national unemployment effect. This implies that the unemployment elasticity does not capture the potential downward effect of national unemployment, which, on average, is more than 16 percent in Eastern Germany during the observation period. It is conceivable in a country where the national unemployment rate is so extremely high, the determination of regional wages is dominated by the national unemployment rate.

Another problem is that the correlation coefficients of the unemployment rate observed at single regions over time are large and diminish slightly over time. They amount to 0.92 for observations one year apart, via 0.86, 0.82, 0.78, 0.73, till 0.65 for observations six years apart. This indicates that dynamic underspecification is likely, even if we would add lag effects, as a result of which the Within, the Between and the OLS estimators produce significant differences among the unemployment elasticity estimate. Important under these circumstances is the variation between the observations at one point in time in relation to the variation within the observations over time for the relevant explanatory variables, in this case the unemployment rate. Total variation of the log of the unemployment rate consists of 70.2% ‘between variation’ and 29.8% ‘within variation’. As the former is much larger than the latter, the preferred estimator of short-run effects should control for regional fixed effects (Within estimator), but not for time period fixed effects (Between or OLS estimator). The choice whether to use the Between or the OLS estimator in this case is simple. As the number of explanatory variables (k=62) exceeds the number of time periods (T=7), it is impossible to compute the Between estimator. This is the reason to table 2. Conversely, the preferred estimator of long-run effects should control for time period fixed effects (Within estimator), but not for regional fixed effects (Between of OLS estimator). We again choose the OLS estimator. If we would choose the Between estimator, the observations would simply be averaged over the observation period, as a result of which the hypothesis whether or not the explanatory variables are correlated with the time period fixed effects cannot be tested. This is the reason to table 3.

Table 2 reports the estimation results when time period effects are not included and regional fixed effects are tested for. Using the test structure denoted in figure 1, we find that FD-2SLS is the preferred estimator, as the unemployment rate does not appear to be strictly exogenous and the regional fixed effects appear to be correlated with the explanatory variables, respectively based on the first and the second Hausman test. In addition to this, we

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12 In Blanchflower and Oswald (1994), the average U.S. and U.K. unemployment rates amount to 6-7% during the observation period.
also find that the estimator should be corrected for spatial effects, as the error terms appear to be spatially correlated. The FD-2SLS estimator fully draws on the time series component of the data and therefore offers the best estimate of the short run unemployment elasticity, which amounts to -0.112 and is statistically different from zero. So we do have found evidence in favor of the wage curve after all. Although this figure is very close to the -0.10 posited in Blanchflower and Oswald (1994), its foundation is completely different. First, it also captures the downward effect of the national unemployment rate. We already saw that this figure would shrink to -0.006 in case this effect would be removed by including time period fixed effects. Second, it is corrected for spatial effects. Under the erroneous assumption that the error terms are not spatially correlated, the unemployment elasticity would more than double at -0.242.

Table 3 reports the estimation results when regional fixed effects are not included and time period fixed effects are tested for. Using the test structure denoted in figure 1, we find that the estimator that controls for time period fixed effects is the preferred estimator. The hypothesis that the unemployment rate is strictly exogenous cannot be rejected, as a result of which instrumental variable methods are not required (based on the first Hausman test). The second Hausman test does not reject the RE model in favor of the FE model, indicating that time period fixed effects are not correlated with the explanatory variables. In addition to this, we find that the FE estimator must be rejected in favor of the FE estimator corrected for spatial effects.

The FE estimator corrected for spatial effects (in default of the RE estimator corrected for spatial effects that, unfortunately, is not available) fully draws on the cross-sectional component of the data and therefore offers the best estimate of the long run unemployment elasticity of pay. This elasticity amounts to -0.042 and is statistically different from zero. It seems that we have found a begin of evidence against the concept of the compensating differential, i.e. people living in high unemployment regions are not compensated by higher wages in the long run. Blanchflower and Oswald have found similar figures of -0.0274 (T-value -2.78) for the U.S. and -0.0385 (T-value 1.61) for the U.K. We should nonetheless be careful. One might argue that regional fixed effects should be included to control for regional differences related to the purchasing power of wages. Regions quoting relatively low wages are often simultaneously characterized by lower cost of living and housing prices, whereas regions characterized by higher cost of living and housing prices are often quoting higher wages as a compensating differential. One argument against regional price deflation is that local government, to provide a higher quantity and quality of governmental services for residents, might tax a significant fraction of a worker’s total earnings. This implies that the inclusion of local taxes in an area deflator could distort the wage rate (Reza, 1978).

13 Although the Hausman tests are not available for the RE and SFD-2SLS estimators corrected for spatial effects, we expect their outcomes to be small. The reason is that the outcomes for these estimators if not corrected for spatial effects are extremely small and the spatial autocorrelation coefficient for those estimators that are corrected within this table appear to be relatively small.

14 Table 4.26, column 1, p.168 and table 6.20, column 1, p.283.
If we would nonetheless rely on regional fixed effects to play the role of regional price deflators, just as in Blanchflower and Oswald (1994, p.101), we would return to table 1, and its unemployment elasticity of pay of -0.006, a figure not statistically different from zero. This shows that the evidence against the long run concept of the compensating differential is rather weak. On the other hand, we may draw the stronger conclusion not to have found any evidence in favor of the long run concept of the compensating differential.

Except for the unemployment elasticity estimates, it is also important to elucidate the spatial autocorrelation coefficient estimates. In short run models, this coefficient appears to be large and significant; see the cases in table 2. The preferred estimator produces an estimate of 0.496 (T-value 8.83). Conversely, in long run models, this coefficient appears to be small and just significant; the fixed effects estimator and the 2SLS estimator in table 3.\(^\text{15}\) The preferred estimator produces an estimate of 0.103 (T-value 2.50). The difference between the short run and long run value of the spatial autocorrelation coefficient can be explained as follows. One of the main characteristics of regional labor markets is that the wage rate and the unemployment rate within these markets tend to follow their national counterparts. Blanchard and Katz (1992) have pointed out that the major part of the regional evolutions of these variables is common to all regions and thus that only a small part is region-specific. In other words, the wage rate and the unemployment rate tend to go up and down together in different regions along the national evolution of these variables over the business cycle. This explains why the spatial autocorrelation coefficient is rather large in typically short run models. In the long run, after the effects of labor supply and demand shocks have settled, the regional wage rate and the regional unemployment return to their equilibrium values. Under these circumstances, the error terms might still be spatially correlated in that neighbouring values, due to unobserved national and sub-national circumstances, are more alike than those further apart. However, this type of spatial correlation is not as strong as its short run counterpart. This explains why the spatial autocorrelation coefficient is rather small in typically long run models.

In addition to this, it can be seen that the spatial autocorrelation coefficient appears to be small when time period fixed effects are included, see the cases in table 1, and to be large when time period fixed effects are not included, see the cases in table 2. It thus seems that the inclusion of time period fixed effects and the use of spatially transformed error terms are two different approaches to deal with the same problem of region-invariant unobserved differences among time periods. However, a closer look at the estimation results shows this not to be the case. Take the unemployment elasticity estimates of the non-spatial models without time period fixed effects in the left column of table 2 as point of departure. Then compare them with the unemployment elasticity estimates corrected for either spatial effects or the inclusion of time period fixed effects, successively in the right column of table 2 and the left column of table 1. In both cases, we see that the magnitude of the unemployment

\(^{15}\) The results produced by OLS and the 2SLS estimators in this table are not really long run estimates, because these estimators do not control for time period fixed effects.
elasticity estimate reduces, but that in the former case the reduction is much smaller than in the latter case. For example, when adopting the estimator that controls for regional fixed effects only the unemployment elasticity amounts to -0.150, when adopting the same estimator corrected for spatial effects it amounts to -0.083, and when adopting the estimator that controls for both regional fixed effects and time period fixed effects it amounts to -0.006. Similarly, when adopting the FD-2SLS estimator the unemployment elasticity amounts to -0.242, when adopting the same estimator corrected for spatial effects it amounts to -0.112, and when adopting the FD-2SLS estimator that controls for time period fixed effects it amounts to -0.010. The difference between the first and the second set of examples is that the in second set the regional fixed effects have been eliminated from the regressions equations by first differencing and that the second set controls for the possibility that the regional unemployment rate is correlated with the disturbance term.

The fact that the reduction when correcting for spatial effects is smaller than when controlling for time period fixed effects can be traced back on the national unemployment rate. The inclusion of time period fixed effects has the disadvantage to eliminate the downward effect of the national unemployment rate, whereas the correction made for spatial effects has not. In conclusion, we may say that the introduction of spatially correlated error terms is an effective method to correct for region-invariant unobserved differences among time periods without eliminating the downward effect of the national unemployment rate.

6. CONCLUSIONS

We have identified two general cases under which one should not control for time period fixed effects to estimate the short run unemployment elasticity of pay. The first case arises when the correlation coefficients of the unemployment rate observed at single regions over time are large and diminish slightly over time. For Eastern Germany we have found that the correlation coefficient between the unemployment rates observed at single regions six years apart still amounts to 0.65. Under these circumstances, dynamic underspecification is likely, even if we would add lag effects, as a result of which the Within, the Between and the OLS estimators produce significant differences among the unemployment elasticity estimate. Applied researchers are often puzzled with these different outcomes. Some of them add time period fixed effects, because conventional tests tell them to do so, and because they are not familiar with that part of the panel data literature that has pointed out that in case the variation between the observations at one point in time exceeds the variation within the observations over time, the preferred estimator of the short run effects should better not control for time period fixed effects. For Eastern Germany we have found a ratio of 2.35 of the ‘between variation’ in relation to the ‘within variation’ with respect to the log of the regional unemployment rate.

The second case arises when the national unemployment rate is rather high. We have seen that wage curve regressions including time period fixed effects measure the impact of
the regional unemployment rate in deviation of the national unemployment rate on the regional wage rate in deviation of the national wage rate. In other words, the inclusion of time period fixed effects has the side effect to eliminate the downward effect of the national unemployment rate. This is problematic especially in countries like Eastern Germany where the national unemployment rate is, on average, more than 16 percent. We have shown that the introduction of spatially correlated error terms is an alternative method to correct for region-invariant unobserved differences among time periods, with the advantage that the downward effect of the national unemployment rate is not eliminated. Using this alternative method, we found an unemployment elasticity of pay of -0.112, a figure almost equal to the standard value of -0.1 posited in Blanchflower and Oswald.

REFERENCES


Hausman-test for FE versus RE

<table>
<thead>
<tr>
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<th>no</th>
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</table>

yes no

Figure 1  Test structure to determine the preferred panel data estimator
Table 1 East German wage curve, 1993-1999. Test structure for regional effects, provided that time period fixed effects are included

<table>
<thead>
<tr>
<th>Model</th>
<th>Non-spatial models</th>
<th>Spatial models</th>
<th>Spatial autocorrelation coefficient</th>
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<tbody>
<tr>
<td></td>
<td>Unemployment Elasticity</td>
<td>Unemployment Elasticity</td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>-0.041 (-6.26)</td>
<td>-0.042 (-6.70)</td>
<td>0.104 (2.52)</td>
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<tr>
<td>Fixed effects</td>
<td>-0.006 (-0.76)</td>
<td>-0.005 (-0.70)</td>
<td>0.046 (1.34)</td>
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<td>Random effects</td>
<td>-0.018 (-2.58)</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>2SLS</td>
<td>-0.056 (-6.76)</td>
<td>-0.057 (-6.86)</td>
<td>0.088 (1.95)</td>
</tr>
<tr>
<td>FE-2SLS</td>
<td>-0.064 (-2.81)</td>
<td>-0.063 (-2.78)</td>
<td>0.024 (0.58)</td>
</tr>
<tr>
<td>FD-2SLS</td>
<td>-0.010 (-0.57)</td>
<td>-0.008 (0.49)</td>
<td>0.045 (1.25)</td>
</tr>
</tbody>
</table>

Hausman tests

<table>
<thead>
<tr>
<th></th>
<th>$\chi^2_{68}$</th>
<th>$\chi^2_{68}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FD2SLS/FE-2SLS</td>
<td>16.3 Not Rej.</td>
<td>61.9 Not Rej.</td>
</tr>
<tr>
<td>FE/RE</td>
<td>125.6 Rej.</td>
<td>NA</td>
</tr>
</tbody>
</table>

$\chi^2_{68}$ = Preferred estimator of the unemployment elasticity when following Blanchflower and Oswald (1994) and Baltagi et al. (2000)

* Test for strictly exogenous explanatory variables

** Test for no correlation between explanatory variables and regional fixed effects

T-values between parentheses, NA not available
Table 2 East German wage curve, 1993-1999. Test structure for regional effects, provided that time period fixed effects are not included

<table>
<thead>
<tr>
<th></th>
<th>Non-spatial models</th>
<th>Spatial models</th>
</tr>
</thead>
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<tr>
<td></td>
<td>Unemployment</td>
<td>Unemployment</td>
</tr>
<tr>
<td></td>
<td>elasticity</td>
<td>Elasticity</td>
</tr>
<tr>
<td>OLS</td>
<td>-0.085 (-5.93)</td>
<td>-0.064 (-5.67)</td>
</tr>
<tr>
<td>Fixed effects</td>
<td>-0.150 (-8.65)</td>
<td>-0.083 (-5.56)</td>
</tr>
<tr>
<td>Random effects</td>
<td>-0.107 (-6.84)</td>
<td>NA</td>
</tr>
<tr>
<td>2SLS</td>
<td>-0.149 (-8.42)</td>
<td>-0.097 (-6.12)</td>
</tr>
<tr>
<td>FE-2SLS</td>
<td>-0.328 (-12.35)</td>
<td>-0.279 (-10.10)</td>
</tr>
<tr>
<td>FD-2SLS</td>
<td>-0.242 (-8.68)</td>
<td>-0.112 (-4.32)</td>
</tr>
</tbody>
</table>

Hausman tests

|                      | FD2SLS/FE-2SLS     | FD2SLS/2SLS    |
|                      | $\chi^2_{62}=435.3$ Rej. | $\chi^2_{62}=266.5$ Rej. |
|                      | $\chi^2_{62}=193.2$ Rej. | $\chi^2_{62}=139.6$ Rej. |

T-values between parentheses, NA not available,

|                      | Preferred estimator of the short-run unemployment elasticity |
|                      | Test for strictly exogenous explanatory variables |
|                      | Test for no correlation between explanatory variables and regional fixed effects |
Table 3 East German wage curve, 1993-1999. Test structure for time period fixed effects, provided that regional fixed effects are not included

<table>
<thead>
<tr>
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<th>Non-spatial models</th>
<th>Spatial models</th>
<th>Spatial autocorrelation coefficient</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Unemployment elasticity</td>
<td>Unemployment Elasticity</td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>-0.085 (-5.93)</td>
<td>-0.064 (-5.67)</td>
<td>0.566 (11.61)</td>
</tr>
<tr>
<td>Fixed effects</td>
<td>-0.041 (-6.29)</td>
<td>-0.042 (-6.70)</td>
<td>0.103 (2.50)</td>
</tr>
<tr>
<td>Random effects</td>
<td>-0.040 (-6.50)</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>2SLS</td>
<td>-0.149 (-8.42)</td>
<td>-0.097 (-6.12)</td>
<td>0.525 (10.79)</td>
</tr>
<tr>
<td>FE-2SLS</td>
<td>-0.054 (-6.64)</td>
<td>-0.056 (-6.84)</td>
<td>0.087 (1.93)</td>
</tr>
<tr>
<td>SFD-2SLS</td>
<td>-0.053 (-6.64)</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

Hausman tests

<table>
<thead>
<tr>
<th></th>
<th>SFD-2SLS/FE-2SLS</th>
<th>FE/RE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \chi^2_{62} = 0.2 ) Not Rej</td>
<td>( \chi^2_{62} = 0.4 ) Not Rej</td>
</tr>
</tbody>
</table>

T-values between parentheses, NA not available

| = Preferred estimator of the long-run unemployment elasticity
| * Test for strictly exogenous explanatory variables
| ** Test for no correlation between explanatory variables and time period fixed effects