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A measurement to analyze the relative change in the Absolute Parity of Power Purchase: An application to the European Union

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Abstract
In the present paper two indexes to measure the changes in the Absolute Purchasing Power Parity (APPP) in the short term of a group of territories that constitute a wider market, using the information of the Harmonized Index of Consumer Prices and the Exchange Rates, is developed. This statistics are used to study the change in relative prices of the countries of the European Union for the period 1996-2002, and the fulfilment of the Relative Purchasing Power Parity (RPPP) theory, taking as a reference the Absolute Purchasing Power Parity in the base year of the Price Index.

Keywords: Purchasing Power Parity, Consumer Price Index, Temporal and Interspatial Comparison.
1. INTRODUCTION

The difficulties found to do these comparisons have given rise to multitude of jobs that can be classified in two large groups. In the first group are the efforts focused to find the best instrument to do comparable the figures of the distinct economies. In this line, it is emphasized the use and the advances in the index numbers theory [for a historic revision applied to price index see Diewert, W. (1993), for a theoretical revision see Balk, B. (1995) and Hill, T. (1988)] and the jobs carried out in the International Program of Comparison (ICP), program belonging to the statistical division of United Nations with near 35 years of life, and whose objective is to produce estimations of the National Gross Product and its components, that can be comparable among countries in real terms. It utilizes the concept of Purchasing Power Parity (PPP), [UN (1992)]. In the same line, the European Program of Comparison is developed by Eurostat and the OCDE since 1980.

In the second group the investigations based in the unit price law and the theory of of the PPP to explain the behavior of the rate of change among two or more countries. In this case, the objective is focused in the analysis of the economic implications of its fulfillment or not, and how being able to test its fulfillment [Cheung, AND W., Lai, K. (2000), Engel, C. (2000), Reads, M. (1976), Taylor, A. (2001)]

This job is inside of the first group of investigations. Two indexes are defined for monitoring changes in the Relative Purchasing Power Parity among a group of countries that do not share a common currency. The field of application for these statistics is limited to those sets of countries which have CPIs elaborated in the same way and that form an overall market with the same or similar rules. An example is carried out using the Harmonized Index of Consumer Prices (HICP), and the Exchange Rates of the European Union countries.

The results show the strong distorting effect that exchange rate has upon the Purchasing Power Parity among countries. The evolution of the internal prices of each country is a residual factor to explain the changes in the Purchasing Power Parity. In this sense, the apparition of euro contributes to a clear stability for the consumers whose
countries have adopted the euro than those that have not adopted yet. However, in the euro zone systematic behaviours in some of the countries are detected that indicate that, their Purchasing Power Parity is changing in a systematic way, with regard to the average Purchasing Power Parity of the Monetary Union.

After defining the Law of One Price, Purchasing Power Parity and the Consumer Price Index, this study proposes, first, a measure by which the APPP of a territory can be directly compared to that of an overall market at any moment of time for which there was a single, same reference moment, and, second, a measure designed to determine whether over the short term changes in the APPP tend to conform to the APPP of the reference period or, on the contrary, tend to diverge from it. In the fifth point an example is developed for the European Union for the period 1996-2002

2. THE LAW OF ONE PRICE, PURCHASING POWER PARITY, AND THE CONSUMER PRICE INDEX

Purchasing Power Parity relates the price of the same product or the same set of products expressed in the same currency for two different territories. That is, if \( p_h^i \) and \( p_h^j \) are the price of the product \( h \) in the territories \( i \) and \( j \) respectively, each in its respective currency, the PPP of territory \( i \) in respect to territory \( j \) for the product \( h \), expressed in the money of an overall market \( G \), which we will call \( PPP_{hij} \), is defined as (1)

\[
PPP_{hij} = \frac{e^{iG} \cdot p_h^i}{e^{jG} \cdot p_h^j}
\]  

(1)

in which \( e^{iG} \) and \( e^{jG} \) are the exchange rates of the currencies of the territories \( i \) and \( j \) with respect to the currency of the general market. That is, the exchange rate of county \( i \) is the quantity of monetary units of the overall market necessary to purchase one monetary unit of country \( i \). Equation (1) can be simplified if we assume \( j=G \). In this case expression (2) is obtained:
\[ PPP_h^G = \frac{e^G * p_h^i}{p_h^G} \]  

(2)

The numerator of (2) is simply the price equivalent of the product \( h \) expressed in the currency of the wider market. If this value is equal to the price of the same article in the wider market, we would say it conforms to the Law of One Price for this product. If the quotient of (2) is equal to 1 for all products, we would say it conforms to the Law of One Price in the joint economy of the two countries \( i \) and \( G \).

However, Purchasing Power Parity is expressed in terms of an aggregate price variable for each economy. In this way, if \( p^i \) and \( p^G \) denote the mean price of a shopping basket in territories \( i \) and \( G \) respectively, the APPP of country \( i \) with respect to the wider market \( G \) is defined as (3).

\[ APPP^G = \frac{e^G * p^i}{p^G} \]  

(3)

If expression (3) equals 1, it is said that it conforms to the Law of Absolute Purchase Power Parity. Obviously, the Law of One Price implies the fulfillment of the Law of APPP, but the reverse does not necessarily hold.

Expression (3) can be expanded by adding a subscript that represents the factor time, obtaining (4).

\[ APPP_{t}^G = \frac{e^G_{t} * p^i_t}{p^G_t} \]  

(4)

The analysis of (4) reveals that if (5) happens, with the value \( k \), which represents the lapse between two moments of time,

\[ APPP_{t}^G = \frac{e^G_{t} * p^i_t}{p^G_t} > \frac{e^G_{t+k} * p^i_{t+k}}{p^G_{t+k}} = APPP_{t+k}^G \]  

(5)

it would either be because country \( i \)’s currency became stronger relative to that of the wider market, or because its prices increased more than those of the wider market, or both. In all three cases, living in country \( i \) would become more expensive in comparison to the wider
market. As a result, people in this territory would be less able to buy goods and services, and thus the standard of living of its inhabitants would have decreased between the moment \((t-k)\) and \(t\) in comparison to the mean situation of the wider market at the moment \((t-k)\).

Here the question becomes how to obtain the value of the \(APPPC^{iG}\), whose calculation requires quantifying the price of the shopping basket in territories \(i\) and \(G\). Studies by Vachris and Thomas (1999), Rao (2001) and Rodriguez, González and Rodríguez (2002) identify some of the difficulties in obtaining the variables \(P^i\) and \(P^G\), for this variable is itself a complex of variables and fundamentally virtual, because it cannot be calculated. The price of a shopping basket can be thought of as a measure of cost of living, but for this its calculation depends on prices, goods and services being homogenous, and they are not. From the economic standpoint, homogenization occurs in value, not prices.

Rodríguez, González and Rodríguez (2002), in their study of price fluctuations using the PPP for a set of territories with a common currency, posed a solution using the statistic defined in (6),

\[
A^{iG}_{(t-k),t} = \frac{I^i_t - I^{i(t-k)}_t}{I^G_t - I^{G(t-k)}_t}
\]  

in which \(I^i_t\) and \(I^G_t\) are the CPI of the territory \(i\) and the wider market \(G\), respectively, at the temporal moment \(t\), both expressed at the same base moment. Recall that CPIs in theory are calculated as the quotient of the costs of two shopping baskets at two moments in the same territory. The moment of the price in the denominator defines the reference period for the CPI. However, the calculation of the CPI does not require calculation of the variable for the price (as opposed to the value) of the shopping basket.

The justification for the use of (6), as well as its interpretation, can be found in Rodríguez, González and Rodríguez (2002).
3. A STATISTIC FOR MEASURING CHANGES IN ABSOLUTE PURCHASING POWER PARITY

When in the wider market territories are present with different currencies, the statistic \( A_{(t-k),t}^{G} \) refers only to the changes produced in Absolute Purchasing Power Parity due to price fluctuations. To include the effect of exchange rates, it is proposed to use the statistic \( B_{(t-k),t}^{G} \) defined in (7), indicating by the subscript 0 the reference moment of the CPIs.

\[
B_{(t-k),t}^{G} = \frac{\frac{\epsilon_i}{\epsilon_0} I_i - \frac{\epsilon_{(t-k)}}{\epsilon_0} I_{(t-k)}^i}{I_t^G - I_{(t-k)}^{G}}
\]  

(7)

The expression (7) will have different meanings according to the value of \( k \). In this section will be considered the specific situation in which \( k=t \). In this situation (7) becomes (8).

\[
B_{0,t}^{G} = \frac{\frac{\epsilon_i}{\epsilon_0} I_i - \frac{\epsilon_0}{\epsilon_0} I_0^i}{I_t^G - I_0^{G}} = \frac{\frac{\epsilon_i}{\epsilon_0} I_i - 1}{I_t^{G} - 1}
\]  

(8)

Based on (8), the following results are possible:

1. If it is greater than 1, country \( i \) has a lower standard of living than the overall market with reference to the CPI base moment.
2. If it is equal to 1, there are no changes in the standard of living of country \( i \) with respect to the overall market with reference to the CPI base moment.
3. If it is less than 1, country \( i \) is less expensive at the moment \( t \) than the overall market with reference to the CPI base moment. This would imply that more goods and services could be purchased, and, in this sense, that the living standards had relatively improved.

Conversely, if the prices in the wider market are not rising, the interpretation of \( B_{0,t}^{G} \) will have a certain indetermination when it takes on negative values.
4. A STATISTIC TO MEASURE RELATIVE PURCHASING POWER PARITY

The statistic proposed in the previous section directly relates Absolute Purchasing Power Parity for any two moments by quantifying its changes. But it does not obtain the value of that parity for any moment, a limitation already explained in the introduction to this study. From $B_{ij}$ one cannot know if the APPPs of the different territories that make up the wider market are compatible with the law of Absolute Purchasing Power Parity. To do so it would have to be seen whether the APPPs of all the territories converge on the value 1.

An alternative strategy is to study Relative Purchasing Power Parity (RPPP). This measure is the relative change among Purchasing Power Parities at two moments of time. Based on the RPPP, it holds that the law of Relative Purchasing Power Parity is fulfilled when (9) occurs.

\[
\frac{\Delta P^G}{P^G} = \frac{\Delta P^i}{P^i} + \frac{\Delta e^j}{e^j}
\]  

(9)

Obviously when the law of APPP is fulfilled, the law of RPPP is fulfilled as well, but the inverse is not true. The challenge posed by the calculation of this measure is once again the calculation of the price variable. One way to resolve it is to use as the rate of price increase the rate of increase in the CPI, since, as (10) shows, they coincide.

\[
\pi_{(t-k),j} = \frac{I_t - I_{(t-k)}}{I_{(t-k)}} = \frac{P_t - P_{(t-k)}}{P_{(t-k)}} = \frac{P_t - P_{(t-k)}}{P_{(t-k)}} = \pi_{(t-k),j}
\]  

(10)

However, Rodríguez, González and Rodríguez (2002) show that for the situation in which there is a common currency, and hence the second term on the right hand part of
equation (9) is zero, fulfillment of the law of RPPP may be incompatible with fulfillment of the law of APPP. The reason for this incompatibility is that the changes are relative to different moments in time. The same problem applies here.

To resolve the problem the statistic $B_{(t-12),t}^G$ is defined as (11), for the case where the frequency of data observation is monthly.

$$B_{(t-12),t}^G = \frac{\frac{e_i^j}{e_0^j} I_t^j - \frac{e_i^j}{e_0^j} I_{t-12}^j}{I_t^j - I_{t-12}^j}$$

Expression (12), an expansion of (11), is used to interpret from the values of $B_{(t-12),t}^G$ whether or not it fulfils the law of RPPP.

$$B_{(t-12),t}^G = \frac{\frac{e_i^j}{e_0^j} P_t^j - \frac{e_i^j}{e_0^j} P_{t-12}^j}{P_t^j - P_{t-12}^j}$$

The statistic $B_{(t-12),t}^G$ is a measure of RPPP, since it measures changes in APPP relative to the single value of the APPP at the reference moment of the index --- whether the change that has taken place in a year in the APPP has lowered, maintained, or increased the APPP in comparison with the situation at the base moment. If $B_{(t-12),t}^G = 1$, it is understood to mean that between the moment $(t-12)$ and $t$ (the period of a year if the observation frequency is monthly), the evolution in country $i$ and the general market has been the same, the initial situation has been maintained, and hence the law of RPPP has been fulfilled in respect to the initial moment. If, on the contrary, $B_{(t-12),t}^G > 1$, it would imply that over the twelve month period country $i$ had become more expensive than the overall market and a citizen of country $i$ would be have a declining standard of living with respect to the wider market. If $B_{(t-12),t}^G < 1$, the interpretation would obviously be the opposite.
In addition to the interpretation of each value of $B^{G}_{(t-12),t}$, the analysis of the series of all the value of $t$ tells whether stable changes are occurring in the APPP. If, for instance, the values of $B^{G}_{(t-12),t}$ systematically hover around 1, above and below it, this would indicate that the APPP is not changing.

If, on the contrary, $B^{G}_{(t-12),t}$ is systematically above 1 it would indicate that country $i$ was steadily becoming more expensive and its RPPP was growing relative to that of the reference moment. Here there are two alternative situations. In the first, the series of $B^{G}_{(t-12),t}$ can oscillate always around a fixed value $\mu$ greater than 1, which would indicate that the APPP has shifted to a new level of equilibrium that can be defined as in (13).

$$APP^G_t = \mu * APP^G_0$$ (13)

The second possibility is that $B^{G}_{(t-12),t} > 1$ but not stable around a single value, which would indicate that country $i$ has not arrived at a situation of equilibrium in regard to the wider market and its APPP is in flux. A similar, but converse interpretation would apply when $B^{G}_{(t-12),t}$ takes values below 1.

5. AN EXAMPLE USING THE EUROPEAN UNION CONSUMER PRICE INDEXES

In the present section evidence about the fulfilment of the Relative Power Purchasing Parity law for the countries of the European Union during 1996-2002 is contributed. Besides, the way of these changes in every country are provided using the two measures presented in the past section, the monthly Harmonized Consumer Price Index calculated by Eurostat and the exchange rates proposed by Werner (2002).

The evolution of $B^{G}_{1996,t}$ statistics, form January 1996 to November 2002 using the average of the year 1996 as the base period, is shown in graph 1. Countries with a more different evolution are remarked.
First of all, the role of two countries outside the Monetary Union, United Kingdom and, in a minor way, Sweden is emphasized. Both countries have strong price stability, so the differences are because of their exchange rate with the Euro. Nevertheless, the exchange rate effect has been different in both countries. Meanwhile United Kingdom is a more expensive country but converging to the rest of the countries of the European Union, Sweden keeps, apart from some months in 1997, with a lower RPPP than in 1996, with a stable evolution.

Secondly, Ireland and Greece are integrated with the remainder of the European Union since the middle of 1998, and Italy since the last months of 1997. The rest of the European Union have values of the $B_{1996,t}^{IG}$ statistic lower than one until 2000, since then, the range of values are reduced to the (2,0) interval.
To analyze the average behaviour over space and time, a distance matrix has been computed among countries and time for the values of the $B_{1996,t}^{G}$ statistic. Euclidean distance between countries is shown in Graph 2, and between two periods of time in Graph 3.

Graph 2: Distance matrix between countries from $B_{1996,t}^{G}$ statistic

As it can be seen from graph 2, the longer distances are for United Kingdom, followed by Sweden, Ireland and Greece.
Graph 3: Distance matrix over time using $B_{1996,i}^{G}$ statistic

Analyzing the main diagonal of the matrix in graph 3 from right to left, it can be seen the convergence process from 1997 to 2002. But two aspects must be remarked: Firstly, the convergence process has not been continuous, and secondly, a divergence process has been produced in 2002.

Moreover, the Euro is achieving, in a certain way, the objective of stability in prices, but it has had significant changes in the APPP of some countries with respect to the rest of European Union, even when these changes have not finished yet. In order to analyze in a deeper way these changes, graph 4 shows the $B_{1996,i}^{G}$ statistics for the countries in the Monetary Union and Denmark for the period between 2000 and 2002.
The most significant fact is that most of the countries are not in a stable position or equilibrium; all the countries have a positive trend but Italy. A more detailed analysis allows us to divide countries in two groups. The first group is formed by Germany, Austria, France, Belgium and Finland. These countries have a value of the APPP between 40% and 80% at the end of 2002 than the one they had in 1996. This implies that these countries are cheaper at the end of the period than in 1996, related to the wider market, the European Union.

The second group is formed by the remainder of the countries. They keep getting more expensive at the end of 2002 compare to the value in 1996. The $B_{1996,i}^{G}$ statistic for Ireland shows an increase of more than 80%, Portugal, Italy and Holland have a value between 20% and 40%. It has to be noted that these percentages are related to the European Union. It means that, when comparing two countries, changes in the PPP for one of them are greater if those countries are from different groups.
For a short term study, and analyze if the European economies are in a convergence process in terms of the Relative Purchasing Power Parity Law, it is more accurate to use the $B_{t-12}$ statistic. Graph 5 shows the values for the $B_{t-12}$ statistics for each of the countries in the European Union, taking as a reference the average value in 1996.

It can be seen four countries with a different behaviour. Two of them, United Kingdom and Sweden, are not in the Monetary Union, and they are not following the RPPP law. The other two countries are Ireland and Greece; both countries show a significant convergence in the APPP changes because of their integration into the European Union. This can be seen from 1999 for Ireland, and from 2000 for Greece.

The effect of the countries outside the Monetary Union over the convergence process in the European Union can be seen in the graph 6. This graph shows the sigma convergence, using the standard deviation for each period of time, for all the countries in the European Union (CE convergence) and those in the Monetary union (UM convergence)

Graph 6: Sigma Convergence in $B_{t-12}$
A more detailed analysis for the $B_{(t-12),t}^G$ statistic for the last period of time allows to classify countries in two groups. Germany, Belgium, Austria, Finland and France are in the first group. These countries are getting cheaper related to the wider market, with values of the $B_{(t-12),t}^G$ statistic under one. On the other hand, Portugal, Greece, Spain, Holland, Italy and Luxemburg are getting more expensive. Ireland is also in this last group, but it has a higher raise in prices. The no convergence process in the last period is due to this double evolution of the two groups.

To evaluate significant differences some tests are carried out. Unit root tests are shown in table 1, tests for the mean of $B_{(t-12),t}^G$ equal to one are shown in table 1. If unit root test is not rejected, it will show changes in the RPPP of that country related to the whole European Union are continuous and in the same way. Under this situation is Germany, Belgium, Denmark, Spain, Holland, Ireland, Luxemburg, Portugal, United Kingdom and the series for the Monetary Union as a whole.

<table>
<thead>
<tr>
<th>Country</th>
<th>Unit Root Test $B_{(t-12),t}^G$</th>
<th>Average of $B_{(t-12),t}^G$</th>
<th>Average $B_{(t-12),t}=1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>Yes</td>
<td>0.30</td>
<td>No</td>
</tr>
<tr>
<td>Austria</td>
<td>No</td>
<td>0.34</td>
<td>No</td>
</tr>
<tr>
<td>Belgium</td>
<td>Yes</td>
<td>0.47</td>
<td>No</td>
</tr>
<tr>
<td>Denmark</td>
<td>Yes</td>
<td>0.99</td>
<td>Yes</td>
</tr>
<tr>
<td>Spain</td>
<td>Yes</td>
<td>1.15</td>
<td>Yes</td>
</tr>
<tr>
<td>Finland</td>
<td>No</td>
<td>0.55</td>
<td>No</td>
</tr>
<tr>
<td>France</td>
<td>No</td>
<td>0.58</td>
<td>No</td>
</tr>
<tr>
<td>Greece</td>
<td>No</td>
<td>1.23</td>
<td>Yes</td>
</tr>
<tr>
<td>Holland</td>
<td>Yes</td>
<td>0.97</td>
<td>Yes</td>
</tr>
<tr>
<td>Ireland</td>
<td>Yes</td>
<td>1.73</td>
<td>No</td>
</tr>
<tr>
<td>Italy</td>
<td>No</td>
<td>1.77</td>
<td>No</td>
</tr>
<tr>
<td>Luxemburg</td>
<td>Yes</td>
<td>0.52</td>
<td>No</td>
</tr>
<tr>
<td>Portugal</td>
<td>Yes</td>
<td>1.27</td>
<td>No</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>Yes</td>
<td>3.44</td>
<td>No</td>
</tr>
<tr>
<td>Sweden</td>
<td>No</td>
<td>0.51</td>
<td>Yes</td>
</tr>
<tr>
<td>Monetary Union</td>
<td>Yes</td>
<td>0.98</td>
<td>Yes</td>
</tr>
</tbody>
</table>
The second group is subdivided into two groups. Firstly, Sweden and Greece have a stable position and it is no rejected that the mean of this two countries are equal to one, so the average RPPP of this countries is constant in the studied period. Austria, Finland, France, and Italy also have a stable position, but the average test equal to one is rejected. This implies that these countries have reached a new RPPP position. Austria, Finland and France have an average of $B^G_{(i-12)i}$ under one, so their prices are getting cheaper related to the wider market. Nevertheless, Italy has an average of $B^G_{(i-12)i}$ above one, so a new RPPP equilibrium is reached and its prices are getting more expensive related to the European Union.

Distance matrix for the $B^G_{(i-12)i}$ statistic between countries, in graph 7, shows that United Kingdom is the country with the most different behaviour, followed by Sweden and, in a minor way, by Ireland, Italy and Netherlands. Moreover, distance matrix over time show a convergence process at the end of the studied period.

Graph 7: Distance matrix between countries using $B^G_{(i-12)i}$ statistic
6. CONCLUSIONS

This study presents two statistics for studying changes in Purchasing Power Parity, that simultaneously take into account spatial and temporal factors, using only information from Consumer Prices Indices and exchange rates. The first statistic, \( B_{GtB}^{IG} \), directly compares changes in APPP for two different moments in time, while the second, \( B_{Gl(t-12)G}^{IG} \), measures changes in APPP with respect to the APPP at the CPI reference moment and indicated whether the law of Relative Purchase Power Parity has been fulfilled for any territory with respect to a wider market. The use of both measures permits the analysis of the comparative evolution of living standards as measured by price changes.

The following conclusions are obtained using the CPI of the countries of the European Union about their changes in PPP:
The use of Euro has given a bigger stability in relative changes in the standard of living of the people in the European Union because of changes in prices.

Most of the countries in the Monetary Union have got cheaper and more competitive during the studied period.

Nevertheless, it seems that a divergence process could be starting in 2002. This divergence is caused not only by the countries outside the Monetary Union like United Kingdom and Sweden; but countries into the Monetary Union are divided in two groups with a different behaviour. Germany, Belgium, AUSTRIA, Finland and France are in the first one, and they are getting cheaper related to the wider market. On the second group, Portugal, Greece, Spain, Netherlands, and Ireland are getting more expensive.

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