The Sources of Economic Energy

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Premise

Why would one think of using Niels Bohr and Werner Heisenberg’s new physics to describe economics phenomena? What on earth has the ambiguous, bizarre and acasual world of “quanta”, which was contested even by Einstein, the father of relativism, to do with the concrete deterministic science of economics? What input could the unverified string theory give, in a context of magnitudes which regard man’s activity such as earnings, savings, investments, capital, costs, revenues and other similar variables which, however, have nothing to do with stars, planets, atoms and subatomic particles?

Whatever the outcomes, I believe, we should feel the same impelling necessity which forced the founding fathers of the science of Economics to draw inspiration from classical physics, Newtonian physics that is, which was universally valid, for planets as well as apples, where everything moves automatically according to a pre-constituted order, according to a mechanism which Smith found useful to describe the invisible hand which governs the economy, and then the Neo-classicists for their elegant universal models, or again to the Darwinists with evolutionary theories where selection is entrusted to the “market” of adaptation or survival. A mechanical or biological system, to research the laws governing the economy, as the economist had no laboratories, he could only rely upon already experimented similitudes.

But the world changes, and it changes at the same speed with which one’s knowledge of it changes. Copernicus’ world destroyed beliefs and superstitions, Newton’s gave certainties, Darwin’s unified life, and Einstein’s opened the universe. None of this has been lost.

In the long walk of history, sometimes slow and sometimes fast, man constantly enriches himself with knowledge. A nation’s wealth is no longer conserved in grain stores or in strong rooms but it nests in the neurons of its inhabitants, the most appreciated products are no longer measured in kilos or metres, but in “recipes” in which the secret of competitive productivity is stored.

The rich have more because they know more. The immaterial economy designs and defines the new territorial boundaries, seeps through the territory, invades, feeds and expands its synapses, ties the knowledge the economy was born and develops from into a diffused memory.

But the economy is deficient in instruments appropriate for observing immaterial things as it also lacks formal models capable of fully interpreting the territory.

The economist knows these gaps well and in order to overcome them is constantly elaborating new concepts, like social capital, new instruments, such as the collective intelligence and others again but which are still not appropriate for synthesizing it into a single formalised expression capable of extrapolating universally valid recipes.
And so the economics of a territory is obliged to struggle along on two equally useful legs, but which force it to take schizoid paths while ignoring each other so as not to fall into obvious contradictions.

The two legs of territorial economics remind us of those of post-Newtonian physics: the General Theory of Relativity, which studies the infinitely big, and quantum mechanics which studies the infinitely small. Although the two theories are both fundamental pillars of Physics, in the light of current knowledge, they should not be equally true, while it is undeniably true that both are at the base of current technology. Einstein dedicated, without success, the last thirty years of his life to solving this “brain-bursting” problem.

Now the new physicists are trying again with the String Theory, above all in the M version or the Theory of Everything. But this fascinating theory does not yet allow us to understand some fundamental things, for example it does not tell us why particles align in a certain way, in a certain order and with a certain potential.

Adapting concepts and paths elaborated by post-Newtonian physics, the economist could do much less and a bit more. Much less because he is not required to solve in any way the mysteries of the universe, a bit more because, perhaps, he can describe without contradictions, using known economic science, what physicists, in their field, are not able to describe: he can tell us, using formal models why at a certain point in time and space a determined productive set composed of a well defined number of “economics quanta” relative to material and immaterial elements, of which is known the magnitude, order and force, behaves like a string and begins to “vibrate” setting off the chain reaction of economic development.

We are dealing, therefore, with an attempt full of ifs, buts and perhaps, ..... but then isn’t that what a researcher’s trade is made up of?

Time, space…

The time and space so loved by philosophers and poets, burden and delight of physicists and astronomers, for a long time have been more for economists elements of inconvenience than of analysis. All this finds a justification in the mechanistic logic which also regulates the great economic theories. In the great machine which moves the Newtonian universe there is no place for chance, for difference, for exceptions: each gear must be governed by general principles which apply to everything: in every place and every time.¹

¹ The variations in rates of growth, in recent years, have been the object of numerous empirical tests and of other attempts at formalisation. Neither the former nor the latter have succeeded in giving a univocal and adequate description of the phenomenon. As regards empirical analyses, the most common methodologies aim at measuring the standard deviation in the distribution of income among regions or the inclination of a straight line of linear regression which links the growth rate to
But the “Newtonian” general economic theory, fascinating though it is and irreplaceable in conferring rigour to theoretical formulations and reducing to a simplified form the apparently (or real) chaos of the great systems, in its necessarily high flying it is unsuitable to interpreting the local level where, instead, it is indispensable to keep one’s feet on the ground, to move in the territory following the infinite combinations of the surrounding countryside, to worm oneself into the maze of economic and social interrelations which make it unique and unrepeatable.

In this intertwining of material and immaterial elements, first the warp then the weave, interact continuously giving life to a variegated and changeable fabric in which the deterministic must constantly and inexorably measure itself against the agent, the prime instrument, the ultimate end of economic activity: man.

“God does not play dice” Einstein used to say,\(^2\) God perhaps doesn’t but men do and men live and operate within spaces in which they accumulate culture, stratify cities, create interpersonal links and relationships, act on the economy by setting up with “their” territory a rapport of reciprocal belonging which maps out an unequal and changeable process of development in time and space.

Nevertheless it wasn’t until the second half of the last century that the conviction became widespread that in order to understand fully some of the economic mechanisms, especially those linked to development, a spatial analysis was unavoidable.\(^3\)

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the level of income. The first methodology, called \(\sigma\) convergence or strong convergence, signals whether regions’ growth rates are tENDING to converge; the second one called \(\beta\) convergence or weak convergence, moves within a convergence logic described by Solow (1956), which tends to verify whether growth rate is higher in countries with lower income.

The outcomes of these analyses have shown deformity in results to the point of rendering the neoclassical formulations of growth models unreliable in reality as they do not allow for growing returns or hypotheses of imperfect markets.

To overcome these gaps, we have seen an upsurge in empirical analyses in which endogenous development is explained with the method of the conditioned \(\beta\) convergence which, by introducing non decreasing sources of return and externalities, permit the realisation of a positive growth rate.

This new methodology (cfr. Mankin N., Romer D., Weil D. “A Contribution to the empirics of economic growth”, *Quarterly journal of economics*, vol. 1992107 pp. 739-774.) in which income growth rate is regressed both on its own level and on territorial indicators, allows us to overcome at an empirical level, the aspatiality of endogenous growth, but does not give univocal results in indicating the trends of compared areas towards results of convergence.

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\(^2\) Physics is deterministic: every cause produces the same identical effect. In quantum theory, instead, as it is the Principle of Indeterminacy which holds true, each cause will *probably* produce the same effect. Einstein, who found the Principle of Indeterminacy unacceptable, repeatedly expressed his conviction that can be synthesized in the famous phrase “God doesn’t play dice” adding, in a letter to Max Born, “in this case I would prefer to be the casino croupier rather than the physicist”.

\(^3\) Systemic studies of the territorial dimension of the economy began with the publication in 1949 of the “The General Theory of Location and Space”, by Walter Isard.
In facing the concept of space the economist has used several different approaches, each of which is strictly linked to a logical scheme, model or system of reference. Basically the concepts of space which are most often found in economics can be summed up as follows:

a) **Physical-metrical space**: this is used by the theory of localisation and is based on physical distance and the relative cost of transport. Space is seen in its merely geographical aspect as a uniform and inert container of economic activity. This type of approach, which deals with localising choices of companies or families, has been used above all to explain how and why, even in a uniform space, productive activities locate in particular portions of territory giving rise to concentrations of economic activity which are the results of contrasting forces exerted by transport costs and by agglomeration economies.

b) **Uniform-abstract space**: this has been used to study the capacity of a local economy to grow and develop by acquiring competitive advantages in respect to other local economies of the same economic system. Within the space thus defined, the conditions of supply and demand are identical everywhere. The space in this type of approach loses physicity and continuity, becoming discontinuous and abstract so

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4 The Italian economy, characterised by unequal regional development, has often been used for this type of study: over forty years of extraordinary state intervention in the South have made Italy the country which has experimented for the longest period with policies tending to reduce the regional growth gap.

The most often applied methodologies have in common a Cobb-Douglas type production function. Among these we notice L. Picci (1977) which relates infrastructure and productivity using the Cobb-Douglas equation estimated in logarithms:

\[ Y_t = f(T, K_t, L_t, G_t, UR) \]

Raffaele Paci and Silvia Saddi (2002) who by operating the logarithmic transformation of the Cobb-Douglas production function

\[ Y = A_t K_{it}^\alpha L_{it}^\beta G_{it}^\gamma \]

estimate the linear function:

\[ y_{it} = a_i + \alpha k_{it} + \beta l_{it} + \chi g_{it} + \epsilon_{it} \]

Lopes (1996) measures the usefulness of expenditure for public works on the levels of private sector production through a Cobb-Douglas function estimated with the method of squared minimals of the type

\[ y_{it} = f(l_{it}, k_{it}, g_{it}) \]

La Ferrara (1999) adds technical progress into the production function

\[ y_t = f(k, l, g, A) \]

and confirms the results through a growth accounting type approach

Ofria (2000) analyses the effects of the lack of infrastructure on private sector productivity of the economy through a production function estimated using the method of ordinary squared minimal, and specified as Cobb-Douglas:

\[ Y = f(K, I, TM, IV, D_t); \]

The results show a strong correlation between public infrastructure and productivity development.

5 The classification is by Roberta Capello [2004].

6 Cfr: W.Alonso,[1974]; R.Camagni,[1999]; Chistaller[1933]; Hoover[1948]; LÖsch [1954]
that it can be used in macro-economic growth models. The space, deprived of the elements of territoriality such as proximity and agglomeration, as well as every other localising characteristic, finds itself emptied of any content which was its own in order to take on the role of receiver of more or less sizeable parts of the national product.\footnote{The ones who adopt this concept of space are the theories and models which refer to the principles of macroeconomics, of neoclassical economics, of the economics of international exchanges, of development economics, giving rise with reciprocal contamination to the neoclassical theory of regional development, to the theory of the basis of exportation, to the theory of factorial endowments; among the most well known models we remember the Harrod- Domar model.} Within this sphere, in order to understand territorial and temporal inequalities of development, the variations in “local” growth rates were put under constant observation applying numerous empirical tests and likewise attempts at formalisation.

But neither the former nor the latter managed to give a univocal and adequate description of the phenomenon, so much so that among economists it is increasingly held that the production function in its classical aspatial form is not capable of fully describing the economic process which it is required to represent.\footnote{“Goodbye production function ... Goodbye neoclassical theory of production ... you ignore space.” Lucio Malfi and Dino Martellato, “Il capitale nello sviluppo locale regionale”, pag. 9, Associazione italiana di scienze regionali. Franco Angeli 2002 Milano.}

c) \textit{Diversified-relational space}: in this type of approach, space loses its essentially geographical-material connotation in order to highlight the immaterial aspect made up of the set of economic-social relationships and governance which are externalised territorially in the form of localisation and spatial proximity economies. Space, thus characterised, becomes a territory subject to its own polarity and specificity: an exogenous source of agglomeration economies which generate economic development. The macroeconomic concept of competitive development, which is characteristic of growth theories which refer to the concept of uniform and abstract space, are here substituted by the concept of endogenous generative-selective development. The analysis moves from the macro level to the micro one to analyse the causes which generate static and dynamic advantages for the economic agents who operate within the local subsystem. The territorial models, which take on board the concept of diversified-relational space, such as those which gave origin to the theorisation of the \textit{industrial district}, of the \textit{milieu}, of the \textit{learning regions}, find within themselves the material and immaterial causes which permit us to select the territory by generating and self-propagating development processes. Relational space is, therefore, also a polarised or diversified space.

d) \textit{Diversified-stylised space}: according to this conception, space preserves diversification, that is the existence of polarity onto which development grafts itself but sacrifices territoriality in favour of stylisation. Space is seen as a set of points at which a cumulative, endogenous and tendentially selective growth sprouts forth due to increasing returns, produced by the local economic system itself, which originate in collective learning processes, economies of scale, and localisation and urbanisation...
economies. Space, even when it is diversified, loses however the very capacity of microterritorial and microbehavioural analysis of the models which assume space as (also) relational.

The formal approach of the macroeconomic-aggregate type if, on the one hand, makes possible, thanks to the use of mathematical approaches relative to the study of the qualitative behaviour of dynamic non-linear systems\(^9\), the inclusion of agglomeration economies in the form of growing returns\(^10\), on the other hand, does not permit us to understand the localised technological externalities and the material and immaterial factors which characterise the space–territory. So space returns to being a simple geographic expression incapable of interacting, as an autonomous added resource, with the other actors in the development process.

The multiplicity of definitions shows, in itself, the “original sin” of the economic theories of space: economic space is never given as a single departure point, with the consequence of making it impossible to have a general unifying theory.

Moreover, even the attempts of genetic engineering, carried out through the inclusion of territorial elements into neoclassical models, with the aim of joining into a single hybrid model the advantages of the theory of \textit{milieu} with those of the theory of engogenous growth, achieves its greatest result in the activation of a process of cross-fertilisation of the two theories. This is certainly useful in the understanding of the differentials of growth rates in “similar” local economies but cannot describe the laws which determine growth, nor can it indicate the limit beyond which the growth rate will stop and congestion diseconomies are produced.

The long journey began with the Solow type neoclassical growth models, characterised by the production function with decreasing returns and with perfect market forms, passing through endogenous growth models, now reaches territorialised forms, which have the advantage of being less abstract than neoclassical models, in that they operate in imperfect markets, but which do not manage to keep the growth rate under control, which is always given as positive. From “implosive” models we pass to “explosive” models.

Forceably including local interrelations into classical production functions is not successful in overcoming the basic contradictions between Newtonian determinism and localistic indeterminism, with the result that the classical elegance is lost without acquiring localistic concreteness.\(^11\)

\(^9\) Chaos Theory; Bifurcation Theory; Catastrophe Theory.


\(^11\) Starting from the statement that the theory of endogenous growth and the theory of the \textit{milieu innovateur} are both based on the idea of endogenous development, Roberta Capello, (2002) an attempt is made to marry up formalised neoclassical models with territorial economic concepts whose spatial variables make every attempt at formalisation complex.

The re-reading in a neoclassical key of the theory of the \textit{milieu innovateur} carried out within models of endogenous growth allows the marrying up of the formalism of economic logic of the neoclassical models with the need to take into the correct consideration the economy’s territorial dimension, characterised by determining phenomena like those of agglomeration and proximity. This happens through the use of mechanisms of realisation of collective learning which, from the
*milieu* theory, are identified in the transfer channels of collective knowledge and in the territorial conditions, which make it possible. According to the *milieu* theory, in particular territorial areas characterised by a high concentration of small specialised firms, the presence of firms with equal productive techniques is possible which, while being exposed to the law of decreasing productivity of single factors, show possibilities of overall increasing development thanks to the effects produced by collective knowledge.

In neoclassical terms all this is translated into the acceptance of the hypothesis of a Cobb-Douglas production function, with decreasing returns in single factors but growing ones at an aggregate level due to the effect of the system’s increased productivity deriving from collective knowledge.

$$Y_t = K^\alpha L^{1-\alpha} h_t^\beta$$

$$H_t = f(mktl; spoff; coopcf)$$

$$y_t/y_t = \alpha k_t/k_t + \beta h_t/h$$

Where: K and L represent physical and labour capital and h (not indexed per single firm) represents the collective knowledge present in the *milieu*, incorporated in the human capital, while mktl is the labour market within the *milieu*, (a labour market which is local, stable and with high internal mobility); spoff represents the spin-off mechanisms of the local firms and coopcf is the cooperation between customers and suppliers.

Growth, as happens in Romer’s model (1986), comes about through a positive externality factor represented in this case by collective knowledge.

A similar attempt is made in Lucas’ model (1988) within a classic Cobb-Douglas type production function. To this end a production function with constant scale returns is used in which human capital is assumed as a source of development.

$$Y_t = a k_t^\alpha (u h_t)^{1-\alpha} h_t^{\phi}$$

where:

- *k* indicates the physical capital per effective labour unit;
- *h* is the human capital defined as the quantity of knowledge held on average by the workers, obtained through education (learning by schooling) or through experience (learning by doing);
- *u* is the time dedicated to work and taken away from study;
- *ϕ* is learning capability, which is taken as linear in respect of the level of knowledge reached. The accumulation of human capital ($\tau > 0$), acquired as an externality, amplifies economic growth but is not indispensable in that collective knowledge (even supposing $\tau = 0$) can arise from processes of involuntary socialisation rather than from cooperative ones, as is theorised in the *milieu innovateur* model.

Human capital is divided into two components: one internal and one external:

$$Y_{it} = a k_{it}^\alpha (Inh_{it})^{1-\alpha} h_{it}^{\phi}$$

where:

- $0 < \alpha < 1$;
- $0 < \beta < 1$;
- $0 < \phi < 1$;
- $0 < \tau < 1$;
- $0 < \gamma < 1$;
- $\alpha + \beta + \tau + \phi > 1$;
- $h_{it} = h_{it}^\beta \phi (1-In)$ (Internal component of human capital)

where with $\beta < 1$. $\phi$ represents the degree of learning of the human capital of the cooperation with local firms where the quota of resources destined to the activity of cooperation is supposed as constant. Thus the *milieu* thesis, of decreasing productivity of knowledge and of the consequent risk of getting trapped in local technological trajectories but to the detriment of endogenous growth, is included even without the characteristic externality of Lucas’ model.

To avoid all this it is necessary to introduce the condition that the local firms have the possibility to be able to draw on unlimited and cost-free external knowledge capable of enhancing their own human capital.
Towards a primary concept of Relational Diversified Stylised Space.

If we cannot use a general theory and at the same time we are not satisfied with dealing on a case by case basis, we must, necessarily, look for new readings. An attempt at reaching that single unifying theory of economic space, objective and myth for all those who study territory, in our opinion cannot but start from the definition of a primary concept of space capable of bringing together the territorial microfoundations of macroeconomic growth models.12

The attempt which we wish to propose here is that of using analogies and contamination with the “new” physics of matter, where even “the dice”, unlike in classical physics, have a role to play. It must not be forgotten that economic activity is an activity in which man establishes the ground rules: by constructing the “dice” and deciding when to throw them.

A concept shared by the various different interpretations of space economy is that of agglomeration, or rather, of a circumscribed and complex concentration made up of heterogenous elements, both material and immaterial, which strongly react together, and, to a lesser and different degree, with the rest of the system in which they are included.

From this common concept an extrapolation of a single, primary definition of economic space could be initiated. One of the first considerations to be made must be that agglomeration is a joint phenomenon due to a time-space activity in that it is the result of the concentration of economic activities accumulated in space in the course of time.

Agglomeration, in its various material and immaterial forms and in its different life cycles, could, therefore, be defined as the result of the time-space interaction of coherent economic phenomena.13

In this view time-space becomes the content and at the same time the single and unifying context in which agglomerations “live”. In relation to an agglomeration space-time is, therefore, both a container, in that it outlines its borders, and the content as it is the essential element of the interaction process which gives body to the agglomeration.

If this type of approach is accepted it is, perhaps, possible to dare to proceed towards interpretations of economic space which could use analogies with post Newtonian physics.14

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12 Capello economia regionale il mulino 2004 pag 29-33
13 Intending as coherent all those happenings capable of developing economic type synergies.
14 According to Einstein’s Theory of Relativity time and space do not have an absolute value but vary in relation to the chosen measurement system, thus the name, theory of relativity. According to this theory relativity is based on two characteristics of light: the first is that light travels through space in the form of “packets of energy” which take on qualities attributable both to particles and to waves; the second is that the speed of light in a vacuum is always constant independent of the movement of its source. From these premises Einstein managed to prove the contraction of time and
The economy as energy.

The first analogy to use is to consider the economy as energy. An economic system is, in fact, a combination of elements which produce “work” in order to make added value. In physics, energy is by definition, everything which is capable of producing work. So an economic system could be described as a combination of energies capable of producing added value.

One real advantage of this type of approach is that it renders the material and immaterial magnitudes, which make up the set of forces which interact in and with the territory under examination, homogenous. Two of the main obstacles which modern localisation economic theory have to face are quantifying concepts which are qualitatively already well defined (like for example social capital, collective learning, knowledge spillover), and outlining territorial spheres like, for example, relational space and proximity, in order to make them includable into formalised systems.

By adopting the similitude economy = energy, the economic fabric could be represented by a set of sources and flows interacting with each other and with the territory from which they rise and/or cross, so that the territory itself becomes a source and flow of energy. If we treat economic energy like light energy we could use some significant experiences derived from quantum mechanics.15

The wave function used by quantum mechanics could, in our case, be configured as an economic energy wave, of internal or external provenance, so that we could say that every territory is crossed by a set of economic waves which characterise it and determine its patrimonial make-up.

space and the increase in mass due to the increase in speed. Another implication of the theory is that light is affected by gravitational pull as an effect of the curvature of space-time. Einstein developed this latter hypothesis on the basis of the non Euclidian geometry of Bernard Riemann, to whom we owe the concept of Unlimited but Finite Space.

15 With the arrival of the atomic era physics discovered that not all the universe is governed by deterministic laws. In the subatomic realm, where atoms liberate or absorb energy only in the form of discreet packets called Quanta, nature does not flow harmonically and does not change gradually with the passing of time but transforms itself in a casual and discontinuous way according to a probabilistic logic. Between 1925 and 1930 the theory of quantum physics was elaborated, which by supplying a consistent description of matter, on a microscopic scale, incorporated the concepts of quantisation and wave-corpusele duality which had floored classical physics. In those first few years two different formulations faced each other: one known as matrix mechanics, - the other as wave mechanics. In the former, to each physically observable quantity a matrix is associated which unlike classical mechanics, obeys a non commutative algebra; while the latter, formulated by Schrödinger in 1923, finds its theoretical basis in waves of matter, formulated by Broglie. Schrödinger himself three years later managed to demonstrate the equivalence between matrix and wave mechanics.
By assuming the economic process to be a process of energy transformation, the economic system could be read as an energy system whose magnitudes are elements of an intermediate state in that they are the result and phase in a continuous change process.

According to this type of approach each economic magnitude can show itself as a stock or as a flow: stock in that it can be a static result of a series of productive phases; flow in that it can be the dynamic input or output of productive processes. The system, in continuous change, incessantly modifies the states of stocks and flows adapting its configuration both as regards typology of productive activities, and as regards their localisations and interconnections.

The more the magnitudes which operate within the system interact in a coherent way, the more the system is efficient. Interaction is coherent when their product makes up a new intermediary state capable of generating new stocks and/or flows.

The systemic coherence allows the single magnitudes to form and act together taking on the characteristics and qualities of a single macro entity.

In the economic system thus defined instrumental magnitudes and autonomous magnitudes operate: that is magnitudes which are manoeuvrable by human intervention, both by firms and states, and self generating magnitudes which are the result of systemic interactions. The former can be considered voluntary, in that the more or less desirable result of direct or indirect intervention but in any case wanted and generated by the economic operator, the latter “involuntary” in that they are present in the economic system due to spontaneous interactions.

Within a thus described system the forces that operate there, both at the kinetic and the static phase, take on, however, localising aspects in that it is a meeting space (arrival and genesis) of instrumental and automatic interactions which contribute to determining coherence and degree of efficiency of the system.

Every spatial macroentity (local economic system) takes on, therefore, its own configuration which derives from more or less efficient interrelations of economic magnitudes of an intermediate state and which manifests itself as an added value of the system.

A thus described economic system read through (some of) the laws of Quantum Mechanics takes on the following characteristics:

1) The evolution of dynamic processes, as a product of instrumental and autonomous actions (the latter being subject to probabilistic logic) makes the whole process indeterministic.

2) The “production function” is defined by discreet rather than continuous functions.

3) The coherence mechanism, if it comes about, will be extended globally to all the economic actors in the studied system as a generator of externality capable of attracting or repelling economic magnitudes in an intermediate state.
The sources of economy

A source is anything which is capable of activating a flow of energy which propagates in the territory. The source is energy in a stock form which gives rise to a flow of energy, it is therefore measurable in its static or dynamic consistency. An economic source can be compared to any other commonly known source: to a star which emits light, or to a glacier which melts. In our case a source is anything which produces economic energy both positive and negative. In the case of externality, for example, a source is an infrastructure in the transport sector like a train station, a port, an airport, a motorway toll station ...; a source can be a school, a hospital, but also a police station, a museum, an industrial zone, ... Attention is therefore addressed both to stock, that is the physical structure and to flow, that is the service that it produces; in our examples respectively, the possibility of using a transport system, education, health care, safety, culture, collective learning, ...

The flows of economic energy expand out into the territory interacting intimately with it and ultimately defining and identifying it. Along their way they meet obstacles which deviate their path, elements which reduce their energy, other flows of a similar nature which by joining together augment their power and others of a different nature which due to interaction will create new forms of economic energy.

Each territory is therefore, characterised by a heterogeneous and omnidirectional set of vector economies, diffuse and interacting among themselves and with the territory, which are generated both inside and outside the territory itself.

As regards externalities, for example, if in the midst of a desert a television satellite during its orbit sends free receivable signals, that tract of desert, for the duration of its irradiation, enjoys vectorial territorial economies equal to the lost cost of a television licence fee. Another example: if we consider an airport, what is taken into consideration for flows, is obviously not the investment cost (a stock element) but, as already stated, the range of action within which its effects are shown. If, for example, point A and point B are both 50 km from the airport, but due to the road network point A can be reached in 30 minutes while reaching point B takes 60 minutes, the vectorial territorial economy which arises in point A is double in respect of point B. Continuing this example it is clear that the vectorial territorial economy generated by the airport is higher (supposing a value scale of +1, -1) and equal to +1 at the airport exit and reduces little by little as the distance, expressed in travelling times, grows. It will reach zero in the areas where the use of airplanes does not allow the accomplishment of any externality at all.

Beyond this threshold the use of aircraft through the airport in question becomes inconvenient: a positive externality becomes a progressively negative one. The externalities we have considered are referable to air transport, but the airport does not only produce primary externalities (strictly referable to its own sector: in this case air transport) in fact, in proportion to its own hierarchical level it also generates secondary externalities both positive (e.g. flying schools) and negative ones, for example environmental pollution (acoustic and from hydrocarbons), in which case
the discussion we presented earlier is overturned in that the costs borne by the
collectivity are inversely proportional to the distance which separates them from the
airport. This does not mean that it can be claimed that the overall result derives from
the algebraic difference between the two effects, in that the externalities do not impact
all economic-social sectors homogeneously.
Continuing with our example, within the range of action of the airport it is probable
that there be vectorial economies generated by schools, hospitals, various services,
productive activities and urbanisation, so that one single territory is generally crossed
by several different vectorial territorial economies. The different vectors weave
themselves in and out through the territory and with the territory, giving rise to an
economic fabric in which the warp and the weft take on different forms and
consistencies which characterise the area and determine its potential supply.
The territory takes on therefore an active function as a generator of its own external
economies and as a user and modifier of external economies of extraterritorial
provenance.

The wave of economic energy.

The wave of economic energy \( \psi_e \) as previously conceived, would have the
following characteristics:
1) it varies with the variations of the territory which it crosses: it determines it and it
is determined by it;
2) it is subject to phenomena of: a) destrengthening; b) strengthening; c) transformation.
   a) There are phenomena of destrengthening due to dispersion and/or absorption.
   Dispersion occurs when part of the energy is used for inappropriate uses (underuse),
   there is absorption when all or part of the energy is used for appropriate uses.
   b) There are phenomena of strengthening when the wave meets waves of a similar
   nature on its path which cause a summing of energies.
   c) There is the phenomenon of transformation when two or more economic waves
   interact generating a new economic wave of a different nature from those which gave
   rise to it.
The constituting elements of an economic wave function are:
Sign: Sources can emit positive economic energy flows \( \psi_e^+ \); or negative ones \( \psi_e^- \)
or as generally happens both together \( \psi_e \).
Magnitude: the quantity of flow is measured at a determined point and at a
determined moment through a scale of values contained within the dominion \([-1 \, +1]\)
in the case of \( \psi_e \); \([1 \, 0]\) in the case of \( \psi_e^+ \); \([-1 \, 0]\) in the case of \( \psi_e^- \). 16
Direction of propagation: in the absence of obstacles the waves propagate in a
circular way diffusing and weakening as they move from the centre towards the
periphery. This merely theoretical and aspatial trend is subject to, at a territorial level,

16 Normalised and standardised values.
the above mentioned phenomena of destrengthening, strengthening and transformation.
The mechanism which generates territorial development can be described in the following way:
The territory is constantly overrun by flows of positive and negative economic energies. Some of these, singularly or interacting with one another, include themselves or generate activating combinations, that is input elements which strengthen or generate new productivities. Each productive unit, old or new, therefore makes up an elementary part of the economic fabric comparable to a cell of a biological tissue or to a structural node of a computer architecture such as an artificial neuron in an artificial intelligence system. 17
Every territory has, therefore, its own patrimony made up of elements of stock and elements of flow. The latter are, if positive, potential activatable resources which become active only if and when included in an entrepreneurial activity (private or public) in a productive combination. If they are negative they represent an element of added cost which acts as a disincentive to development.
In the one case as in the other, great spaces for intervention open up for economic policies which will be able to act by promoting initiatives intended to resolve potential territorial technological paradigms and/or remove the causes which slow down development.
One determining and discriminating element of development is the occurrence, within the territory considered, of an optimal number of “qualified activating combinations”. The accomplishment of this is due to the occurrence of casual and governed elements. Casualty is due to the critical mass of the economy, so that the more agglomeration factors it presents, the more cases of possible activating combinations will increase. Governed elements depend instead on voluntary acts which are due, above all, to economic policy intervention.
The architecture of nodes, which sums historical stratifications and successive modifications, and the circulation of flows as they evolve and reconfigure themselves is conditioned by casual elements, as described, for example, by the theory of cumulative circular causation, 18 and by governed elements such as those due to private initiative and to economic policy intervention.
In conclusion, territorial development depends on chance, on casualty and on human intervention.
From this we can ascertain that:
■ The probability of activating combinations increases with territorial and relational proximity.
■ Relational proximity produces only causal combinations.
■ Territorial proximity produces causal and casual combinations.
■ Territorial interrelational combinations represent the territorial supply of externality.

17 To our end the second similitude is preferable for obvious reasons of calculation.
Economic activity occurs when a precise combination of supply corresponds to a likewise punctual demand combination.

Territorial supply is discreet: it moves in jumps, so combinations over or under those activating represent a waste of resources.

Territorial supply selects development.

The territory therefore takes on an active function as a generator of its own external economies and as a modifier of external economies of extraterritorial provenence.

Quanta and economic strings.

If we consider a very large number of independent identical systems, each formed by a fraction of economic energy: Quanta of the economy \( Q_e \) which moves under the action of a given external force, the set can be described as a single wave function which contains all the information about it\(^{19}\).

Therefore economic space would be “filled” by a heterogeneous set of Quanta of economic energy \( Q_e \) which opportunely combined (in quality and quantity) would...

\(^{19}\)\( \psi_e (x,y,z,t) \).

The probability of finding the particle in a determined territory of the economic system, or rather in the element of volume \( dr = dx dy dz \) around the point \( r = xyz \) at the time \( t \) is

\[
P(r,t) dr = |\psi_e (r,t)|^2 dr
\]

So the density of probability of position of the quantum of economic externality \((Q_e)\) on each of the systems is given by:

\[
P(r,t) = |\psi_e (r,t)|^2 = \psi_e *(R,T) y (R,T).
\]

If one possible state of a set of identical systems is described by the wave function \( y_1 \) and another state of the same set by a wave function \( y_2 \) a linear combination

\[
y = c_1 y_1 + c_2 y_2
\]

is a wave function which describes a possible state of the set given by the overlap of the wave functions with \( c_1 \) and \( c_2 \) as constants.

If \( y_1 \) and \( y_2 \) are expressed in the form

\[
y = |\psi_e_1| e^{i \alpha_1} = |\psi_e_2| e^{i \alpha_2}
\]

the modulus of the square of \( y \) is given by:

\[
|y|^2 = |c_1 \psi_e_1|^2 + |c_2 \psi_e_2|^2 + 2Re \{ c_1 c_2^* \psi_e_1 \psi_e_2 \} e^{i (\alpha_1 - \alpha_2)}
\]

in general \[
|y|^2 \neq |c_1 \psi_e_1|^2 + |c_2 \psi_e_2|^2
\]

The wave function \( \psi_e (r,t) \) can be calculated from an equation of the differential to the partial derivatives called Schrödinger’s Equation.

This equation must be linear and homogeneous in order to satisfy the conditions imposed by the principle of overlap and of the first order in the temporal derivative of \( \delta t \), to satisfy the hypothesis that the system’s evolution must be totally determined by knowledge of the wave function in any given moment.

A formal description could be given by associating a Hamiltonian to each element of economic energy with an auto value which depends on its characteristic parameters. The whole system could be characterised by a pair interaction.
give rise to productive processes which can be represented through a “territorial”
production function: that is expressed as a combination of material and immaterial
Quanta of the economy ($Q_e$), active in the territory.
Resorting to another analogy of the new physics, every “territorialised” production
function could be represented by a *Productive String* ($S_p$) whose elements are the
Quanta of economic energy ($Q_e$).\(^{20}\)
One result of the mathematics of strings is that the theory requires more than three
dimensions so the further dimensions can only be “seen” through mathematics and
then be “translated”, as far as possible, in lower dimensional analogies.\(^{21}\)
We also adopt this type of approach imagining that hyperspace, if the good Lord does
not mind, is similar to a hypermarket.
The hypermarket is visited every day by a certain number of customers each of whom
brings with them a shopping list. If, for example, our customer is a chef, on the basis
of the various different recipes he intends to follow, he will select, among the many
varied goods, only those foodstuffs which are found in his recipes. The same thing
happens in our imaginary hyperspace: the subject who intends to start any productive
activity moves around among the various sales counters which display “Quanta” of a
heterogenous nature, searching for “economic quanta” which are part of the technical
formula for the production he intends to undertake.
But the Quanta hypermarket is a very special hypermarket as it is possible to
purchase, besides the elements of the material economy, also elements of the
immaterial economy produced in and from the territory.
Returning to the example of the chef, not only can he choose what to offer to eat but
also in what area he wants to set up the restaurant through the choice of
*Environmental Quanta* ($Q_A$) present in the territory. Entrepreneurial action, technical
information, economic and environmental Quanta, therefore, belong to one and the
same space which we could call economic isospace. It is in the field of *economic
isospace* that the productive strings, made up of material and immaterial elements of

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\(^{20}\) String Theory is a theory of quantum mechanics which includes all forces and all matter. The
time, which, up until now, has had no direct experimental support, harmoniously unifies quantum
mechanics and general relativity, the laws of the very small and the very large, which are otherwise
incomprehensible.

\(^{21}\) The concept of Hyperspace derives from a science-fiction application of the Black Hole Theory,
in fact the first theorisation of hyperspace on a scientific basis could be attributed to Robert
Heinlein who, through Starman Jones, the hero of the novel of the same name, compares space to a
scarf on which, at a certain distance, Mars and Jupiter are reproduced: if we fold the scarf so that
Jupiter is directly below Mars the distance between the two planets becomes minimal; “our space, he
adds, can be screwed up to make it fit into a coffee cup ... if you exceed the speed of light where
space folds up and is congruent, you fall back exactly in the same space ... but at a great distance.
How far depends on how space is folded” (Robert Heinlein, Starman Jones. Milano, Nord, 1989).
Einstein and Rosen take the example to a scientific level. (A. Einstein; Nathan Rosen “The Particle
the economy, in the form of economic energy Quanta, interact incessantly modifying themselves and modifying the economic system to which they belong. The economic isospace is, therefore, a dimension of the hyperspace where interactions between energies which produce economic events happen. The economic isospace is intimately and inseparably linked to the territory because it identifies and determines it in all its parts. It identifies it in that the nature and the intensity of the interactions is neither uniform nor continuous so that each part of the territory has its own peculiarity within a single system. It determines it in that every economic phenomenon originates from an exchange of energy between forces which interact in the territory and with the territory. The territory is, at one and the same time, the source of its own energy, a transit zone for external energies and a possible interaction zone between internal and external energies which combine in production processes in the wide sense of the term.

Simplifying to the extreme, we could imagine that every productive process is characterised by a String \((\tilde{S}_p)\) in which every productive factor, identified in time and space, vibrates under the weight of market forces seeking its state of highest efficiency.\(^\text{22}\)

According to this virtual representation of the economic system every element which contributes to the realisation of any product, in our case every Quantum of active economy \((Q_e)\) inserted into a productive String \((\tilde{S}_p)\) is subject to continuous variations due to modifications of the technological paradigm and/or the quality/price relationship in the market of the factors. An innovation, for example, could bring, according to its importance, both a partial modification of the string structure, through quantitative and/or qualitative substitutions both of input and/or output, and the total elimination-inclusion of strings due to radical changes in technology and/or products.

The Strings \((\tilde{S}_p)\) with their incessant reformulation are, therefore, both actors in the life cycle of the product and dynamic elements of the system, which under their impulse undergo a constant continuous evolution.

**Immaterial Economics, environmental Strings and neural Networks.**

Recently, the neural network has also been used in the elaboration of spatial data, which, using technology based on Artificial Intelligence, is capable of giving “\textit{a posteriori}” behavioural rules for development processes. What is new compared to traditional models, is above all its capability to elaborate, starting from a data base, a

\(^{22}\) The string or vibrating cord is the essential element in String Theory whose postulate states that the fundamental ingredients present in nature are not point particles with a zero dimension, but rather miniscule one-dimensional filaments called Strings whose vibrational structures dictate the properties of particles and the types of force at work in the world.
bottom-up type inductive process capable of training the process units which make up the network’s distributed architecture.\textsuperscript{23}

Neural logic, in our opinion, lends itself well to creating models of the dynamics of immaterial economies\textsuperscript{24} represented by externality Quanta (Q_έ) whose characteristic is that of not following a deterministic logic.

The contamination, or if you like, the multidisciplinary contribution of cutting edge research in the field of neuroscience and quantum mechanics, could turn out to be interesting in trying to re-read a phenomenon which is determinate and evasive, such as that of the role of externalities in the local development of the economy.

Among the several studies which have dealt with the relationship between quantum coherence and biological systems, the one by Penrose (1994) seems to us the most interesting.\textsuperscript{25}

R. Penrose hypothesises that, in man, the passage from the pre-conscious state to the conscious one happens when the cerebral tubules reach the state of global coherence, as also happens with electrons in superconductivity.\textsuperscript{26}

If we consider the cognitive process as the path along which thoughts and actions are produced, and the economic process as the path along which you achieve the production of goods and services, the cerebral tubules could be compared to the productive units of an economic system. Thus doing, the results reached by the theory of quantum coherence would not be dissimilar to those which are found in an economic system, in that the dynamic processes would be regulated by a logic which is not rigorously deterministic and the “coherence” effect would be extended to the whole system. At this point we need to ask ourselves what the energy in an economy could be that would “cool” the economic system down enough to allow superconductivity so as to reduce a productive process’s resistance (costs) to a minimum. The hypothesis which we propose is that of assigning this role to the positive externalities (Q_έ\+)	extsuperscript{27}.

It must be noted that, unlike the physical system, economic externalities could be introduced not only from outside, but also produced by the territorial system itself.

\textsuperscript{23} Among the new spatial models, those based on Cellular Automata take on particular importance in that they are characterised by the definition of a web of cells and neighbouring relations. Although they have the advantage of being formed by dynamic algorithms, capable of representing a vast range of situations and processes, they are limited to using a top-down procedure which needs “a priori” rules which are not capable of identifying nor yet finding the rules which regulate its dynamics.

\textsuperscript{24} see social capital and others ...

\textsuperscript{25} Quantum coherence is that physical mechanism by which a great number of particles act together taking on the characteristics and qualities of a single macro-entity, like for example the emission of Laser rays or superconductivity.

\textsuperscript{26} Metals taken to low temperatures show the phenomenon of superconductivity, that is they do not pose resistance to the passage of a current. This is due to the electrons which transport the electric current which, at very low temperatures, move all together in a coherent way. It is the cold which produces the coherency in the system and exaggerates its efficiency.

\textsuperscript{27} Negative economic externalities (Q_έ\−) would increase the resistance inside the system with a consequent increase in costs.
Thus a process of auto coherence would be established which could activate a process of virtuous development of an endogenous type as happens, for example, in industrial districts.

This type of approach, using A.I. technology, such as Neural Networks, could help us to understand the rules which govern the passage from pre-development to development (pre-consciousness to consciousness) where contingent elements can always show up including accelerating or braking elements.

If we compare the sources in the economy to artificial neurons the links between the economic sources can be represented by synaptic connections whose terminals (unlike the natural ones) can emit both exciter signals and inhibitory ones. In the first case they will be positive externalities and in the second they will be negative externalities. The matrix of the synaptic connections, in this case, will be represented by the matrix of the activating combinations activating Strings.

In order to transmit a gradation of signals of varying intensity which can be usefully adopted by receiving neurons, we could use a continuous linear activation function; where $K$ is a constant and the function can be forced to operate within a certain interval $[0,1]$ or $[-1,+1]$ to contain the activation of the neuron. We could also use non-linear continuous type functions like the “sigmoid” or “logistic” one:

$$
\Phi (A) = \frac{1}{1+e^{-Kr}}
$$

28 An artificial neuron can be defined as a set of synapses which correspond to the terminals of other neurons, by a threshold and by an activation function.

$$
A_i = \sum_j W_{ij} X_j - \nu \quad \text{(Activation potential)}
$$

$A_i$ = net input or activation potential; $W_{ij}$ synaptic weight; $X_j$ = entrance signal; $\nu$ = threshold value.

The activation potential subjected to the action of an activation function will determine the neuron’s response $y_i$;

$$
y_i = \Phi \left( \sum_j W_{ij} X_j - \nu \right);
$$

The activation function determines the type of response that a neuron is capable of emitting. A neural network is composed of several neurons each of which receive one or more synaptic connections: if we analyse the system in vectorial notation, given that a neuron’s activation potential is a linear function of entrance signals, the activation potential of a whole stratum of neurons $A^T = A_1, A_2, A_3, \ldots, A_n$ can be rewritten as

$$
A = WX
$$

That is, the product between the entrance signals vector $X^T = (X_1, X_2, \ldots, X_n)$ and the matrix of the synaptic connections where the lines m correspond to the receiving neurons and the columns n to the entrance signals $W = (w_{11}, w_{12}, \ldots, w_{mn}, w_{21}, w_{22}, \ldots, w_{mn})$.
In our case it is thought that auto-associative networks are preferable in that they possess a single stratum of units wholly connected among themselves so that each unit receives input both from outside and from other internal units. Auto-associative networks are particularly good for learning, storing and reconstructing patterns as they are capable of associating the different parts of a single pattern one to another.\textsuperscript{29} Learning comes about by presenting each pattern to all the other units of the network and modifying the values of all the synaptic connections. When the pattern has been memorised, it is enough to present only a part and calculate cyclically the output of each node

$$y_i = \Phi(I_i + \sum_{j \neq i} W_{ij} y_j)$$\textsuperscript{30}

where each neuron-string does not possess auto connections and it is possible to use threshold values.

The network, through interactions of the activation calculation, is capable of reconstructing the original version of the memorised pattern. The activation values of the strings (nodes), which depend on the type of function applied, could be restricted within a certain domain $[0,1]$ or $[-1,+1]$. If we suppose the use of continuous activation strings within the domain $[0,1]$ the input information, in presenting to the network entrance nodes the various different activating combinations, could be carried out using a coding distributed in such a way that many units contribute to representing each single object.

The use of this type of coding, unlike local coding, does not imply the use of all the nodes to represent all the memorised activating combinations in a complete way. The saving in nodes is not the only reason which leads us to believe distributed coding is advantageous rather than local coding. The latter, in having to use as many units as there are activating combinations which are intended to be represented in the network, not only requires knowing in advance the number of activating combinations in order to predispose the corresponding number of units, but it also appears to be very fragile in that the occasional loss of a unit (economic quantum) implies the loss of the corresponding combination (STRING).

Without getting into the different types of distributed coding, whose choice depends on what type of operation the network has to carry out on the entrance information, we could advance the hypothesis of using a distributed coding with partially overlapping fields. This would give a coarse coding in which every entrance node would be activated in function of the contributions of the single externality sources falling within the various receiving fields. The capacity of the neural code to distinguish between the various characteristics of the flows of externalities $a$, as

\textsuperscript{29} The architecture of a neural network is characterised by the distinction between entrance neurons and exit neurons, by the number of strata of synapses and by the presence of neurons of retroaction, these latter being called also recurring connections. The networks are generally divided into hetero-associative networks, where the entrance nodes are distinct from the exit nodes, and into auto-associative networks.

\textsuperscript{30} Where $I_i$ is the external input of the node $i$ and $j$ is the index which refers to the other network units.
happens in an artificial retina, depends on the range of each zone \( r \) and on the number of zones on which they have an effect \( n \).

\[ a \propto nr \]

Another way of being able to represent a sufficiently elastic and robust binary code could be obtained by codifying the flow characteristics (waves) of the externality sources. Each entrance unit would codify the presence and the value of a certain characteristic so that each activating combination would be defined by the combination of active units in the network.

**Normalisation of vectors.**

Real entrance data can be put into a continuous cycle, using recording apparatus, or predefined lists. In both cases the data will not be homogeneous so that it will be necessary to normalise them by bringing the length of each vector back to 1: each component of the vector will be therefore divided by the vector norm:

\[
\| X \| = \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2}, \quad x_i = \frac{x_i}{\sqrt{\sum_{j=1}^{N} x_j^2}}
\]

to classify the activating combinations, many models can be used; in the case of a binary or bipolar neuron the classification is carried out by learning to activate itself only in the presence of a determined input group and to remain inactive in all the other cases. If there are more than three connections they will identify a hyper-plane of separation in the multidimensional input space.

John Hopfield clarified the similarities between some concepts of statistical mechanics and the functioning of a neural network made up of simple interconnected bipolar units (McCulloch-Pitts neurons) using the concept of “energy” to describe the dynamics of the network’s functioning. Among the various elaborations of Hopfield’s network with stochastic units, particular importance is taken on by Bolzmann’s Machine which can be defined as a general model for recurring stochastic neural networks with symmetrical connections.\(^{31}\)

Bolzmann’s Machine can be used to carry out, among other things, tasks of memorising and optimising thanks to the possibility of presenting each pattern (in our case each activating combination) to all the network’s visible units; when instead it has to do classification or approximation tasks, the input vector is applied contemporarily to the output units, too.

---

\(^{31}\) The main difference between Hopfield’s network with stochastic units and Bolzmann’s Machine is that the latter, in addition to the visible units, can also have hidden units usable exclusively by the internal elaboration.
Bolzmann’s Machine can take on any architecture as long as the connections are symmetrical $W_{ij} = W_{ji}$. One fundamental characteristic of Bolzmann’s Machine is that it works in terms of probability of the system’s state: the learning of a determined pattern (each activating combination – potential string: activatable string) corresponds to the maximisation of the probability that the visible units are in that state and to the reduction in probability that they be in other states: through this process, the modification of synaptic weights makes it so that the states of the network’s visible units take on the “probability distribution” of the input-output vectors of the training group.

**Models based on competition mechanisms.**

In the case of externality sources, by using competition models not all the activating combinations will be effectively activated, in that within the system a competition will start which will activate one (or more) winning units. *Active strings*

Each activating combination *Activable strings* will occupy a precise position in respect of the others to which a precise territorial allocation will correspond.\(^{32}\) The learning mechanism in these cases will be of a post-synaptic type so that only the weights of the active units are modified on the basis of a Hebbian rule. The winning unit or activated combination will be $i^*$ the one which holds the potential of activation $A_i$ of the highest intensity

$$A_i = \sum W_{ij} x_j$$

for that determined activating combination (input pattern x).\(^{33}\)

As more than one unit is active, for any determined pattern, the more the vector of synaptic weights is similar to the input pattern, if we normalise the weights, the smaller is the Euclidian distance between the synaptic weight vector $W_i$ of the winning unit $i^*$ and the input vector $x$ in respect to all the distances between the synaptic weights of the other units and the input vector itself.

$$\|W_i - x\| \leq \|W_j - x\| (\forall j)$$

So, the choice of the winning unit can be obtained both by normalising all the synaptic vectors. In this case the index of the winning unit will correspond to the unit with the maximum activation $i^* = i \max (\sum W_{ij} x_j)$ and will be found through the unit which has the minimum Euclidean distance between the input vector and its own synapses vector

\(^{32}\) It has lateral connections.

\(^{33}\) Clarify what is meant in this potential case of higher intensity activation.
At the end of each cycle of the competition process, the network will, therefore, be capable of configuring the active units which will be inserted into the economic system taking on the role of new sources. It is on this type of mechanism that Kohonen’s maps are based where learning tends to develop a set of synaptic vectors which optimally represent the distribution of the input vectors. In order to reduce enormous quantities of data, a distribution of vectors is subdivided into similar groups and, for each group, a representative vector is identified. The criterion of optimality of the set of representative vectors (code) is given by the error of quantisation. The final synaptic weights of a Kohonen map make up the distribution code of the training vectors where each of them represents the prototype vector of a class of input vectors. The advantage of using this self-organised process of classification in respect to supervised algorithms is that even incomplete data can be used for training.

This way of proceeding is very useful when dealing with real data, having, for example, to classify firms on the basis of product cycles or technological matrices, in that it could be difficult to have the same amount of data for all the firms so that some vectors would present empty elements. Kohonen’s maps are capable of making up for

\[
i' = i \min \left( \sum_j \left( x_{ij} - w_{ij} \right)^2 \right)
\]

---


35 Similarly to what happens in vectorial “quantumisation”.

36 Average deviation between the original vector and the one reconstructed with a reconstruction code.

37 To avoid distortion of the map due to the missing uniformity of the probability density of the input vectors, several variations on the algorithm have been developed based on a redefinition of the competitive process and of the synaptic weight rule. For all [ S.Haykin Neural Networks: A comprehensive Foundation, Upper Saddle River, NJ, Prentice Hall. 2. ed 1999]

38 In the case of incomplete data, the winning unit corresponds to the highest value of the sum of the quadratic deviation between the components of the input vector and the synaptic vector calculated in the subspace defined by the available elements

\[
i' = i \max \left( \sum_{j \in P_t} \left( x_{ij} - w_{ij} \right)^2 \right)
\]
these deficiencies by supplying correct classifications in any case using the final weights to find the value of the missing elements.
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