THE DYNAMIC OF POVERTY IN GERMANY 1985-1995: WHICH FACTORS INFLUENCE THE DURATION OF A SPELL OF POVERTY?

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Abstract

Since the seventies most of the research which quantifies poverty and inequality of income has been carried out in the short-term using an annual household budget survey. Recently, the availability of Panel data has facilitated the appearance of longitudinal studies. The objective of our research is to carry out a dynamic analysis of poverty and to know which factors influence the duration of a spell of poverty. The data we will use is from the Paco Panel Comparability Project. This data was collected and elaborated by the Human Capital and Mobility Programme.

Key words: Poverty, longitudinal analysis, non-parametric methods.

Introduction

Cross-sectional data sets have been used in poverty studies for a long time, enabling, among other things, the proportion of poor households to be quantified and the socio-economic characteristics of this population to be identified at any particular time. The current development of another type of data set, panel data, has made a detailed analysis of the phenomenon of poverty over time possible. Those surveys allow us to determine if poverty is either a short-term situation, due to negative factors such as depression in the economic cycle, or if it is a long-term situation, that is, if it is a permanent condition which can be transmitted from one generation to another. The distinction between “transitory poverty” and “permanent poverty” is vital nowadays, above all in industrialised countries, in which a large number of households suffer short spells of poverty during their lives. Furthermore, this distinction is
crucial for the formulation of public policies, since the transitory poverty appears as a phenomenon which tends to solve itself, whereas the permanent poverty constitutes a more serious problem whose reduction requires direct action of specific public policies.

The objective of our research consists of an analysis of the dynamics of poverty in Germany covering the period from 1985 to 1996, using non-parametric analysis. This paper is composed of three sections and two appendices. In the first section, we describe the data set and those methodological aspects for the quantification of poverty. In the second section, after introducing the basic concepts of duration, a non-parametric analysis is carried out. In the third section, we present some conclusions and future lines of research. And lastly, two appendices explain the process followed in order to obtain durations of spells of poverty and a brief development of the Kaplan-Meier estimator is included.

1. - Methodological aspects in the measurement of poverty and the Paco data set

Data set

We are going to use the data set PACO (the Panel Comparability Project) in our research. This is a cross-sectional and longitudinal data set. It has variables which are comparable for several countries (Germany, United Kingdom, France, Luxemburg, United States, Hungary, Poland). These variables were created from the original panel data of each country (PSELL/Luxemburg, BHSP/UK, ESEML/Lorraine, SOEP/Germany, and PSID/USA), using a common plan and were adjusted by using an international standardised classification in those cases where available. Although this data set allows us to carry out comparisons between the aforementioned countries, our interest is focused on an analysis of poverty, its duration, together with those factors which influence duration, for the case of Germany. This country was selected for its longer period of observation of our panel data.

Unity of analysis

The units of observation in the data set PACO for Germany represent the individual and the household. A household consists of all people who live together in a dwelling unit (house, apartment, group of rooms or single room). People within a household may be related to each other. Included are unmarried couples, if the couple living together has a fairly permanent arrangement. A household may consist of more than one family, if the people are related to
each other. Excluded are lodgers, conventional roommates, or employees who share the housing unit. The unit chosen for this study has been the household.

Indicator

Many papers have been discuss whether poverty should be expressed in terms of income or expenditure. We prefer the disposable income variable because it allows us to measure the household’s opportunities, and therefore those of their members to accede a worthy life. Hence, we are interested in that concept of poverty relating to the right of minimum resources. Furthermore, the information on household expenditure is not available in the data set PACO.

Measurement of poverty

From a general perspective the measurement of poverty consists of two different properties, Sen (1976, 1978, 1979):

1. - Identification of the poor population (Who are the poor?)

2. - Aggregation (How can the characteristics of poverty of different households be combined in an aggregate measure?)

The process of the identification includes the use of equivalence scales and the construction of poverty lines, although there is no a general agreement among researchers about what constitutes a poverty line and an adequate equivalence scale.

Equivalence Scales

Households differ in size, composition, age of their members, place etc., and the household’s necessities vary according to these characteristics. Therefore, it is necessary to take these differences into consideration in any analysis of poverty. In accordance with our objective we have chosen the most straightforward equivalence scale, the per capita scale. Despite being the simplest, it is at the same time, the most rue since it only takes the household size into account, which gives the per capita income. Nevertheless, if we define the poverty line as a percentage of the mean/median of the per-capita income, the results of poverty are very similar to those using different equivalence scales.

Poverty line
We are going to use a relative poverty line by defining the threshold as 50 percent of the mean/median of disposable per capita income for each year. Relative poverty lines constitute widely accepted tools for poverty analysis in developed countries. Indeed, the poverty line defined as 50% is shown as the standard poverty line in the European Union.

Once the poverty line is calculated, the problem of aggregation appears, that is, How is it possible to combine the characteristics of poverty of different households in one single measure? Two indices of poverty have already been extensively used to measure the degree of poverty in a community.

1. - The incidence of poverty, $H$, the Head Count ratio, gives us the proportion of the population that is found below the poverty line, $H = q/N$, where $q$ is the number of poor and $N$ is the total population.

1. - The intensity gap ratio $I$, $I = \sum_{i=1}^{q}(z - y_i)/z$ where $z$ is the poverty line given and $y_i$ is the disposable per capita income of the $i$th household. This index can be interpreted as the percentage of average income which would be necessary to place the poor household at the poverty line $z$, which is the level of subsistence.

Sen (1976, 1978, 1979), criticized both measures using the following arguments:

1. - The index $H$ is not sensitive to the degree of poverty of the poor, since the distance from the poverty line is not taking into account. Furthermore, this index is not sensitive to any transference of income when the number of poor remain unchanged after this transference.

2. - On the other hand, the index $I$ is not sensitive to the number of poor involved in the process of the aggregation. Nevertheless, it is sensitive to any transference between poor people, when the number of poor remains unchanged.

However, they are both frequently-used indices when a first view of the situation of poverty in a society is sought.

The family of Foster, Greer, and Thobercke indices constitutes a good alternative to the indices above. The Foster, Greer and Thobercke indices weight the importance of the poverty gap in relation with the threshold.

$$FGT_\alpha = \frac{1}{N} \sum_{i=1}^{q} \left( \frac{z - y_i}{z} \right) ^{\alpha - 1} \quad \alpha > 0$$
As the parameter alpha increases, the index gives more weight to the larger poverty gap. Hence alpha is interpreted as the parameter of the aversion of poverty. It can be observed for $\alpha=1$, the FGT index coincides with the index H, and for $\alpha=2$ the index FGT coincide with the product HI. The Foster-Greer-Thobercke index satisfies the transference axiom for $\alpha>3$, therefore the FGT$_4$ index is frequently used when the aim is to pay more attention to the poorest.

In this research we are going to use indices H, FGT$_2$ and FGT$_4$, to obtain an initial idea of the path of relative poverty in Germany over 1985-1996.

**Figure 1: Path of the index “Head-Count-Ratio”, H=FGT$_1$, Germany 1985-1996**

In Figure 1, it is possible to observe the trajectory followed by the “incidence of poverty” calculated using two poverty lines: i) the poverty line defined as 50% of the mean of the per-capita disposable income (pob media) and ii) the poverty line defined as 50% of the median of the per-capita disposable income (pobme). Using the second poverty line, we can see that from 1987 to 1989 the proportion of poor households decreases, peaks sharply in 1990, and from 1992-1996 increases gradually to reach a value in 1996 very similar to that as in 1985. As could be expected according to the statistical properties of the mean and the median, the proportion of poor obtained with the mean is greater than that calculated with the median.

**Figure 2: Path of the indices FGT$_2$ y FGT$_4$, Germany 1980-1996**
As we can observe in Figure 2, the path followed by \( \text{FGT}_2 \) and \( \text{FGT}_4 \) over the period 1985-1996 is very similar to that followed by the index \( \text{FGT}_1 \). The greatest proportion of poor households are found to be in the years 1985 and 1990, and furthermore these households were more intensively poor. Lastly, the intensity of poverty increases smoothly from 1991 to 1996.

2. - Statistical analysis of the durations in the state of poverty

Basic concepts of durations

Statistical analysis of the duration models, also called survival models, has been applied in different branches, acquiring a great apogee in Medicine and Engineering. Its application to social sciences began towards the end of the 70’s and, during the last 15 years has increased in researches which has analysed issues such as, the period of time an individual is unemployed, the duration of a patent, the duration of marriage etc..

Duration models allow us to analyse the transition of an individual between an initial and final state. In these models the key variable is the duration which can be defined as the period of time an individual remains in his/her initial state and ends when the individual moves to the final state. In our case, the initial state is the situation of poverty and the final states corresponds with slipping out of poverty. The duration of a household in poverty which is a random variable will be called \( T \). This is a continuous variable, which takes values in the interval \([0, +\infty)\), taking zero at the moment when the household slips into poverty and, furthermore for simplicity it is supposed, that \( P(T=+\infty)=0 \).
Our analysis covers the period 1985-1996, as a consequence, a common characteristic to the duration data appears in the form of censoring. Possible schemes of censoring are right-censoring and left-censoring. Right-censoring takes place in our example when the household still remains in poverty once the period of study ends. Left censoring appears when the households were already in poverty before the period of study begins.

In the context of duration models, certain functions are of special relevance. These include the density function $f(t)$, the distribution function $F(t)$, the survival function, and the hazard function (risk function or exit probability) of the variable $T$.

The survival function of variable $T$, denoted by $S$, is defined as:

$$S(t) = P(T > t) = 1 - F(t)$$

This function gives us the probability of the duration in the poverty state being greater than time $t$. It is a non-increasing function, and takes the value one in $T=0$ and the value zero in $T=\infty$.

The hazard function of the variable $T$, denoted by $h$, is defined as:

$$h(t) = \lim_{\Delta t \to 0} \frac{P[t \leq T < t + \Delta t / T \geq t]}{\Delta t}$$

$h\Delta t$ is interpreted as the probability that a household still remains in poverty at moment $t$ and goes out of poverty just after $t$.

The specifications of the risk and survival functions determine the model in a unique way, as do the density and the distribution functions. The main relationships between these two functions are:

$$h(t) = -\frac{\partial \log(S(t))}{\partial t}$$

$$S(t) = \exp\left(-\int_0^t h(x)\,dx\right)$$

$$f(t) = h(t)\exp\left(-\int_0^t h(x)\,dx\right)$$

*The empirical analysis*
Once we have looked at the main concepts in order to analyse the durations, we apply them to our data set.\(^1\). In general, the methods applied to the durations can be classified as parametric or non-parametric. In the first case, it is not necessary to specify the shape of the distribution which generated the data, whereas it is necessary for a non-parametric methods. Our objective is to carry out a non-parametric method thereby leaving parametric analysis for later research.

Table 1 displays a description of the structure of permanence in poverty according to the number of years in the sample.

\[
\begin{array}{cccccccccc}
T. povery & dj & wj & nj & S(t^+) & E.S. & inferior L. & superior L. & \theta = dj/nj & ES(\theta_j) \\
0 & 0 & 0 & 5884 & 1.0000 & & & & & \\
1 & 1536 & 3033 & 5884 & 0.7390 & 0.0077 & 0.7503 & 0.7278 & 0.2610 & 0.0057 \\
2 & 305 & 417 & 1315 & 0.5676 & 0.0170 & 0.5868 & 0.5489 & 0.2319 & 0.0116 \\
3 & 113 & 170 & 593 & 0.4594 & 0.0262 & 0.4836 & 0.4364 & 0.1906 & 0.0161 \\
4 & 64 & 73 & 310 & 0.3646 & 0.0391 & 0.3936 & 0.3777 & 0.2065 & 0.0230 \\
5 & 21 & 53 & 173 & 0.3203 & 0.0482 & 0.3521 & 0.3294 & 0.1214 & 0.0248 \\
6 & 11 & 36 & 99 & 0.2847 & 0.0599 & 0.3202 & 0.2532 & 0.1111 & 0.0316 \\
7 & 4 & 14 & 52 & 0.2628 & 0.0720 & 0.3027 & 0.2282 & 0.0769 & 0.0370 \\
8 & 3 & 9 & 34 & 0.2396 & 0.0896 & 0.2857 & 0.2010 & 0.0882 & 0.0486 \\
9 & 0 & 9 & 22 & 0.2396 & 0.0896 & 0.2857 & 0.2010 & 0.0000 & 0.0000 \\
10 & 2 & 6 & 13 & 0.2028 & 0.1484 & 0.2712 & 0.1516 & 0.1538 & 0.1001 \\
11 & 0 & 5 & 5 & 0.2028 & 0.1484 & 0.2712 & 0.1516 & 0.0000 & 0.0000 \\
\hline
Mean & & & & & 4.68 & 0.12 & 4.44 & 4.92 & \\
Median & & & & & 3.00 & 0.12 & 2.77 & 3.23 &
\end{array}
\]

In this table \( nj \) is the number of households at risk at \( T_j \) (that is, the number of households whose period of poverty is greater than or equal to \( T_j \)); \( dj \) is the number of households which leave poverty at \( T_j \); \( wj \) is the number of right- censored data at \( T_j \) (that is, the number of households, for which the duration in poverty is not known with any accuracy, since either the time of the study has run out before the households have slipped out of poverty, or because they have been lost for the sample). The only information available for these second group is that their period of permanence in poverty is greater than \( T_m \). The estimation of the survival function and the hazard function is obtained with these three statistics. The Kaplan-Meier estimator is used to estimate the survival function. Furthermore, the standard error and an asymptotic interval of 95% of confidence has been provided using the Greenwood formula. In the same way, an estimation of the hazard function and its standard error is provided. In the last rows the estimations of the mean (limited to a period of 11 years) and the median, are given, including their standard errors and an asymptotic interval of 95% of confidence. A brief explanation is shown in Appendix II.

\(^1\) See the appendix
Figure 3 is a the graphical representation of the Kaplan-Meier estimation.

We can see that if a household is in the first year of a spell of poverty, the probability of exiting is 0.26. However, if a household is in the sixth year, the probability of exiting falls to 0.111. The exit rate declines for either of two reasons: Firstly, we have negative duration dependence from the first period, that is, as goes on the probability of slipping out of poverty decreases. And a second reason could be that the probability decreases as a consequence of the heterogeneity of the poverty population.

We are going to see how some characteristics of the households affect the process of moving into and out of poverty. The way of analysing this effect consists of comparing survival and risk function of the unit of analysis with different characteristics. These variables can be categorized into two groups: i)
Variables related to the main breadwinner such as gender, the level of education, and the work situation. As these last two variables vary over time, we have adopted that category at the moment of slipping into poverty. ii) The other group of variables to be analysed deals with the characteristics of the household. In particular, we will see the effects that the number of income recipients and the household size have on the duration. As in the first case, we measure these variables at the moment of slipping into poverty.

In Table 2 the effect of the gender variable on the durations of poverty is analysed.

**Table 2: Durations in the sample according to the gender of the main breadwinner**

<table>
<thead>
<tr>
<th>Poverty years T.</th>
<th>dj</th>
<th>wj</th>
<th>ηj</th>
<th>S(t+)</th>
<th>E.S.</th>
<th>ηj</th>
<th>ES($)$</th>
<th>dj</th>
<th>wj</th>
<th>ηj</th>
<th>S(t+)</th>
<th>E.S.</th>
<th>ηj</th>
<th>ES($)$</th>
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<td>0</td>
<td>3846</td>
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<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>2038</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1010</td>
<td>1972</td>
<td>3846</td>
<td>0.7374</td>
<td>0.0096</td>
<td>0.2626</td>
<td>0.0071</td>
<td>526</td>
<td>1061</td>
<td>2038</td>
<td>0.7419</td>
<td>0.0131</td>
<td>0.2581</td>
<td>0.0097</td>
</tr>
<tr>
<td>2</td>
<td>199</td>
<td>270</td>
<td>864</td>
<td>0.5676</td>
<td>0.0210</td>
<td>0.2303</td>
<td>0.0143</td>
<td>106</td>
<td>147</td>
<td>451</td>
<td>0.5675</td>
<td>0.0242</td>
<td>0.2590</td>
<td>0.0200</td>
</tr>
<tr>
<td>3</td>
<td>69</td>
<td>107</td>
<td>395</td>
<td>0.4684</td>
<td>0.0312</td>
<td>0.1747</td>
<td>0.0191</td>
<td>44</td>
<td>63</td>
<td>198</td>
<td>0.4414</td>
<td>0.0479</td>
<td>0.2222</td>
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<tr>
<td>4</td>
<td>52</td>
<td>51</td>
<td>219</td>
<td>0.3572</td>
<td>0.0490</td>
<td>0.2374</td>
<td>0.0288</td>
<td>12</td>
<td>22</td>
<td>91</td>
<td>0.3832</td>
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<td>69</td>
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<td>0.0363</td>
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<tr>
<td>Median</td>
<td>3</td>
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<td></td>
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<td></td>
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<td>0.17</td>
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</table>

The comparison between the survival function of the household according to the gender of the main breadwinner is shown in Figure 4.
In principle, according to the data from the table above it is possible to conclude that there are no differences in the duration of poverty between those households whose main breadwinner is a man and those households whose main breadwinner is a woman. Therefore, the phenomenon of the feminization of poverty obtained with the cross-sectional data cannot be translated to longitudinal studies. This result, however, should be clarified, since the non-existence of significant differences can be due to the effect of other variables. In fact, it is assumed that both sub-samples are homogenous with regard to other variables which can affect duration. This assumption, however does not appear to be correct, as has been verified with some homogeneity tests. We will have to rely on parametric or semi-parametric methods for a rigorous analysis on how the gender of the main breadwinner can affect the duration in poverty.

In Table 3, the results of the comparison of the sub-samples are found according to the level of education of the breadwinner at the beginning of his/her poverty spell. The graph representation of the survival function can be found in Figure 5. The education level is composed of the following categories: i) ED1, First-level (primary), 1st to 4th grade. ii) ED2, Second level (first stage), which corresponds to the obligatory education; iii) ED3, Second level (second stage), which includes preparation for University or level equivalent not directly leading to a profession and technical or vocational education leading to occupation or group of occupations and apprenticeship. iv) ED4, Third level which includes University, technical College or Institute education,

Figure 5
According to Table 3, it is possible to state that the greater the level of education is the shorter durations in poverty are. So, for instance, whereas 37.46% of the households whose main breadwinner with obligatory education (ED2 level) have persistent spells of poverty, this percentage decreases to 30.81% in the case of ED3 level of education and for a 27.06% for the university and college graduates.
When defining the duration as the period of permanence in poverty, if we compare two survival curves, the one placed below the other signifies that the group represented by this curve will remain less time in poverty in comparison with that group defined by the curve located above. In other words at any time the estimated proportion of individuals who continue being poor is less for the group represented by the curve below than that represented by the curve situated above. When analysing the sub-samples of the different levels of education, we can observe that in general, the survival curve for those households whose main breadwinner has higher education, is below the other curves. We can also see that the estimations of the risk function for those households where ED4 is 1, tend to be greater than in the other three sub-samples, even though it is possible to talk about another behaviour. It seems that the risk function must be increasing, and hence it must have positive dependence duration.

<table>
<thead>
<tr>
<th>First level of education</th>
<th>Second level of education</th>
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<tbody>
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</tr>
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<td>0</td>
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</tr>
<tr>
<td>1</td>
<td>70</td>
</tr>
<tr>
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</tr>
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<td>8</td>
</tr>
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<table>
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<table>
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<th>Third level of education</th>
<th>Fourth level of education</th>
</tr>
</thead>
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</tr>
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<tr>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
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<tr>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Median</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.00</td>
<td>4.63</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Median</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.00</td>
<td>4.06</td>
</tr>
</tbody>
</table>

When comparing two survival curves, the one placed below the other signifies that the group represented by this curve will remain less time in poverty in comparison with that group defined by the curve located above. In other words at any time the estimated proportion of individuals who continue being poor is less for the group represented by the curve below than that represented by the curve situated above. When analysing the sub-samples of the different levels of education, we can observe that in general, the survival curve for those households whose main breadwinner has higher education, is below the other curves. We can also see that the estimations of the risk function for those households where ED4 is 1, tend to be greater than in the other three sub-samples, even though it is possible to talk about another behaviour. It seems that the risk function must be increasing, and hence it must have positive dependence duration.
The results of the comparison of the sub-samples according to work situation are presented in Table 4, and the graphical representation of the Kaplan-Meir estimator in Figure 6.

Table 4: Durations in the sample according to the work situation

<table>
<thead>
<tr>
<th>Work situation, Others</th>
<th>Work situation, Employee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poverty T. dj wj nj S(t+) E.S. θ</td>
<td>dj wj nj S(t+) E.S. θ</td>
</tr>
<tr>
<td>0 0 0 1630 1.0000</td>
<td>0 0 3171 1.0000</td>
</tr>
<tr>
<td>1 362 1010 1630 0.7779 0.0132 0.2221 0.0103 0.788 222 684 0.5735 0.0236 0.2368 0.0163</td>
<td></td>
</tr>
<tr>
<td>2 70 74 258 0.5669 0.0402 0.2713 0.0277 162 222 684 0.5735 0.0236 0.2368 0.0163</td>
<td></td>
</tr>
<tr>
<td>3 23 33 114 0.4525 0.0619 0.2018 0.0376 57 78 300 0.4645 0.0366 0.1907 0.0226</td>
<td></td>
</tr>
<tr>
<td>4 11 14 58 0.3667 0.0887 0.1897 0.0515 35 37 165 0.3660 0.0366 0.1907 0.0226</td>
<td></td>
</tr>
<tr>
<td>5 4 13 33 0.3222 0.1098 0.1212 0.0568 10 25 93 0.3266 0.0653 0.1075 0.0321</td>
<td></td>
</tr>
<tr>
<td>6 3 5 16 0.2618 0.1627 0.1875 0.0976 6 21 58 0.2929 0.0791 0.1034 0.0400</td>
<td></td>
</tr>
<tr>
<td>7 1 1 8 0.2291 0.2105 0.1250 0.1169 3 9 31 0.2645 0.0986 0.0968 0.0531</td>
<td></td>
</tr>
<tr>
<td>8 0 2 6 0.2291 0.2105 0.0000 0.0000 2 4 19 0.2367 0.1261 0.0000 0.0000</td>
<td></td>
</tr>
<tr>
<td>9 0 1 3 0.2291 0.2105 0.0000 0.0000 0 5 13 0.2367 0.1261 0.0000 0.0000</td>
<td></td>
</tr>
<tr>
<td>10 0 0 0 0.2291 0.2105 0.0000 0.0000 0 2 2 0.1775 0.2399 0.2500 0.1531</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>Limited to10</td>
</tr>
<tr>
<td>Median</td>
<td>3.00</td>
</tr>
</tbody>
</table>

As can be appreciated the survival function of the retired group is below the others. This fact shows us that in Germany the pensioners enjoy a good social protection system, which reduces the duration of any spell of poverty. Differences between the sub-samples of employees and the unemployed are also observed, above all for those periods greater than five years. Whereas the probability of remaining in poverty five years or more is 0.30704 for the unemployed sub-sample, for the employee sub-sample is 0.3266. However, it seems there is no major difference in behaviour over short periods. Non parametric analysis is not adequate for this characteristic since the work situation varies over the period. As before, a parametric or semi-parametric analysis which allows the introduction of time-varying covariates would be more suitable.
The results of the comparison of sub-samples according to the number of income recipients are shown in Table 5, and the graphical representation of the survival function in figure 7.

As can be deduced, the number of income recipients of the household is a variable which influences the duration of permanence in poverty. It can be observed that the more income recipients there are, the lower the survival function is, hence the greater the number of income recipients the shorter the time of permanence in poverty. In this way, for instance, the percentage of households with...
persistence periods was 35.3% for the sub-sample with one single income recipient, whereas for the sub-samples of two and three or more income recipients are 20.16 and 19.85% respectively. It can be seen that the risk rates are higher for those households where the number of income recipients is greater, and negative dependence duration is also observed within the sub-samples.

Table 5: Durations of poverty according to the income recipients of the household

<table>
<thead>
<tr>
<th></th>
<th>One recipient of income</th>
<th>Two income recipients of income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dj</td>
<td>wj</td>
</tr>
<tr>
<td>Poverty T.</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1115</td>
<td>2849</td>
</tr>
<tr>
<td>2</td>
<td>213</td>
<td>333</td>
</tr>
<tr>
<td>3</td>
<td>88</td>
<td>128</td>
</tr>
<tr>
<td>4</td>
<td>43</td>
<td>55</td>
</tr>
<tr>
<td>5</td>
<td>17</td>
<td>43</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>27</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>5</td>
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<tr>
<td></td>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>Mean</td>
<td>5.01</td>
<td>0.15</td>
</tr>
<tr>
<td>Median</td>
<td>3.00</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Three or more income recipients of income

<table>
<thead>
<tr>
<th></th>
<th>One recipient of income</th>
<th>Two income recipients of income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dk</td>
<td>wj</td>
</tr>
<tr>
<td>Poverty T.</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>70</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>23</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
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<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
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<td>0</td>
</tr>
<tr>
<td>9</td>
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<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Mean</td>
<td>3.55</td>
<td>0.46</td>
</tr>
<tr>
<td>Median</td>
<td>2.00</td>
<td>0.46</td>
</tr>
</tbody>
</table>

We have also carried out the analysis for the sub-samples according to the number of members of the household. The results appear in Table 6 and the graphical representation of the Kaplan-Meier estimation for the different sub samples in Figure 8.
According to the data, it is not possible to infer with clarity that the number of members of the household influence the time of permanence in poverty. There is no systematic pattern for the different survival functions since they continually change their positions for the different sub-samples according to the durations. This may be due to the variable household size interacting with the number of income recipients, and it is very possible that the different sub-samples are not homogenous with regard to the number of income recipients, which leads to the confusing influence of the household size on the duration of poverty. It would be necessary to choose a parametric or semi-parametric analysis with covariates which control the heterogeneity in order to understand how the household size influences on the permanence in poverty.
<table>
<thead>
<tr>
<th>Poverty T.</th>
<th>dj</th>
<th>wj</th>
<th>nj</th>
<th>S(t+)</th>
<th>E.S.</th>
<th>Dj</th>
<th>wj</th>
<th>nj</th>
<th>S(t+)</th>
<th>E.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household with one single member</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1375</td>
<td>1.0000</td>
<td>0</td>
<td>0</td>
<td>1625</td>
<td>1.0000</td>
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<tr>
<td>Mean</td>
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<td>904</td>
<td>1375</td>
<td>0.7942</td>
<td>0.0137</td>
<td>0.2058</td>
<td>0.0109</td>
<td>479</td>
<td>851</td>
<td>1625</td>
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<td>188</td>
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<td>0.0375</td>
<td>0.1862</td>
<td>0.0284</td>
<td>69</td>
<td>97</td>
<td>295</td>
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<td>0.0774</td>
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<td>0.0513</td>
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<td></td>
<td>4</td>
<td>7</td>
<td>6</td>
<td>32</td>
<td>0.3823</td>
<td>0.1214</td>
<td>0.2188</td>
<td>0.0731</td>
<td>9</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>2</td>
<td>5</td>
<td>19</td>
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<td>0.1447</td>
<td>0.1053</td>
<td>0.0704</td>
<td>7</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0</td>
<td>4</td>
<td>12</td>
<td>0.3421</td>
<td>0.1447</td>
<td>0.0000</td>
<td>0.0000</td>
<td>3</td>
<td>7</td>
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<tr>
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<td>0</td>
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<td>8</td>
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<td>0.1447</td>
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<td>0.0000</td>
<td>1</td>
<td>0</td>
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<tr>
<td></td>
<td>8</td>
<td>0</td>
<td>2</td>
<td>6</td>
<td>0.3421</td>
<td>0.1447</td>
<td>0.0000</td>
<td>0.0000</td>
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<td>2</td>
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<tr>
<td></td>
<td>9</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>0.3421</td>
<td>0.1447</td>
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<td>0.0000</td>
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<td>4</td>
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<tr>
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<td>1</td>
<td>3</td>
<td>0.2280</td>
<td>0.4331</td>
<td>0.3333</td>
<td>0.2722</td>
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<tr>
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<td>1</td>
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<td>0.4331</td>
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<td>0.0000</td>
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<td>Household with three members</td>
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<tr>
<td>Median</td>
<td>5.25</td>
<td>0.36</td>
<td>4.6</td>
<td>0.25</td>
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<td>3</td>
<td>0.19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Household with five or more members</td>
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</tr>
<tr>
<td>Median</td>
<td>4.48</td>
<td>0.25</td>
<td>4.68</td>
<td>0.26</td>
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<td></td>
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<td></td>
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<tr>
<td>Mean</td>
<td>3.00</td>
<td>0.24</td>
<td>3</td>
<td>0.18</td>
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<td></td>
<td></td>
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</tbody>
</table>
3. Conclusions

The objective of this research has been to understand how to apply a longitudinal analysis for the study of the dynamics of poverty. Focussing on the data set for Germany, we have checked how a non-parametric analysis is sufficiently illustrative about the experience in poverty of a group of households. Thus, it has been possible to determine that the probability of being in poverty is greater than a determined number of years and therefore, to obtain the proportion of households in transitory and permanent poverty. We have observed that there are relevant variables clearly related with the household and with the main breadwinner to explain the permanence in poverty, such as the level of education, the work situation of the main breadwinner and the number of income recipients in the household. However, this type of analysis insufficient, because, as we have mentioned in our research, the significant differences between the survival functions can be due to the heterogeneity of the sub samples for each value of the variable of interest. Furthermore, some of these variables do not remain unchanged over a spell of poverty. Hence, we would like to continue the research using a parametric or semi-parametric analysis which allows the introduction of time-varies covariates. And lastly, it would be interesting to apply this methodology to the case of Spain and to carry out comparisons.
Appendix 1. Obtaining spells of poverty

It has been vitally important for the development of this study, the calculation of the spells of poverty for which the following steps have been taken:

1.- Calculation of the poverty line and classification of the households into poor and non-poor

The definition of disposable income which has been used to calculate the poverty line and to classify the households into poor and non-poor is:

Total disposable income = total gross income\(^2\) - (contributions to Social Security + income tax)

The following table displays the number of households with information available for the income variable for each year. The value of the mean and median of the per-capita disposable income used in the definition of the poverty line is also shown. In particular, the poverty line has been chosen as 50\% of the median of the per-capita disposable income.

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Observ.</td>
<td>5053</td>
<td>4831</td>
<td>4771</td>
<td>4571</td>
<td>4445</td>
<td>6472</td>
<td>6358</td>
<td>6326</td>
<td>6298</td>
<td>6442</td>
<td>6605</td>
<td>6525</td>
</tr>
<tr>
<td>Mean</td>
<td>1453.77</td>
<td>1576.54</td>
<td>1586.35</td>
<td>1687.72</td>
<td>1800.30</td>
<td>1474.82</td>
<td>1670.31</td>
<td>1716.75</td>
<td>1912.70</td>
<td>1961.04</td>
<td>2006.40</td>
<td>2005.43</td>
</tr>
<tr>
<td>Median</td>
<td>1200.00</td>
<td>1325.00</td>
<td>1350.00</td>
<td>1430.06</td>
<td>1531.63</td>
<td>1195.00</td>
<td>1359.04</td>
<td>1487.18</td>
<td>1591.67</td>
<td>1654.21</td>
<td>1700.00</td>
<td>1702.08</td>
</tr>
</tbody>
</table>

2.- Linking of households and calculation of durations

As a consequence of the rotation of the households in the sample in the period observed, it has been necessary to link the households to see their changes of poor/non-poor states and in this way to calculate the spells of poverty. According to the transitions suffered by the households, we have classified the households into four types: i) non-poor households during the whole period, that is, those households with no spells of poverty; ii) households that were poor at for least one year over the period 1985-1996; iii) households that were always poor over that period and therefore suffered right- and left-censored spells of poverty with 12 months duration, and iv) households with no available information of the income variable. The number of households of each type appears in Table 8. For our analysis those households of type i), iii) and iv) were eliminated from the sample.

\(^2\) For a more detailed analysis of the components of the total gross income, see the user’s manual (version 30-9-97).
Table 8: Household distributions 1985-1996 (N=11,148)

<table>
<thead>
<tr>
<th>Never poor</th>
<th>escape from poverty some year</th>
<th>Always poor</th>
<th>Non available</th>
</tr>
</thead>
<tbody>
<tr>
<td>3273</td>
<td>5282</td>
<td>1400</td>
<td>1193</td>
</tr>
</tbody>
</table>

3.- Calculation of the spells

With the sub sample of households ii), we have calculated the duration of each spell and the type of censoring. Several spells of poverty were obtained for some households. The kind of spells and the number of them are shown in the following table. Non-censored spells and right-censored spells were used for our empirical analysis.

Table 9. Distribution of the spells of poverty and censoring

<table>
<thead>
<tr>
<th>Non Censoring</th>
<th>Left-censoring</th>
<th>Right-censoring</th>
<th>Left-and right-censoring</th>
</tr>
</thead>
<tbody>
<tr>
<td>2059</td>
<td>1404</td>
<td>3825</td>
<td>1702</td>
</tr>
</tbody>
</table>

Appendix II. The product limit estimator (PL) of the survival function

Let us suppose that we have a sample of N spells without censoring and have K ≤ N periods of permanence in poverty ranked according to their magnitude, \( t_{(1)}, t_{(2)}, \ldots, t_{(M)} \). Let \( d_j \) be the number of individuals who leave poverty at \( t_{(j)} \), where \( d_1 + d_2 + \ldots + d_M = N \). Defining the survival function as

\[
S(t) = P(T \geq t), \quad t \geq 0,
\]

we can use \( \hat{S}(t) \) as the estimator of this the empirical survival function. \( \hat{S}(t) \) is defined as the proportion of the N individuals leaving at or after t. Doing some operations and re-ranking the terms the empirical survival function can be written as

\[
\hat{S}(t) = \prod_{t_{(j)} < t} (1 - \hat{\theta}_j),
\]

where

\[
\hat{\theta}_j = \frac{d_j}{N - d_1 - d_2 - \ldots - d_{j-1}} = \frac{d_j}{d_j + d_{j+1} + \ldots + d_M}
\]

which gives us the proportion of individuals who leaves at \( t_{(j)} \) with regard to the total number of households which could leave the poverty state. Therefore the probability of leaving is estimated at \( t_{(j)} \) given that it has survived until this date, that is, the risk of leaving is estimated at that moment, \( \hat{\theta}_j \). If
we divide $[0,t]$ into $k+1$ intervals such as $[0,t_{(1)}], [t_{(1)},t_{(2)}],..., [t_{(k)},t_{(k+1)}]$, where $t \in [t_{(k)},t_{(k+1)})$, we observe that this last expression is based on the idea that the probability of surviving until $t$ can be rewritten as the product of probabilities of surviving in each interval previous to $t$, given that it survived at the beginning of each interval.

Let us define the risk set $N_j$ as the set of individuals who have not left poverty observed at the moment just before $t_{(j)}$ and let us denote the number of elements of this set as $n_j$. Hence, as can be verified the denominator of the quotient of $\hat{\theta}_j$, that is, $N-d_1-d_2-...-d_{j-1}$, is equal to $n_j$. Therefore, we will have

$$\hat{S}(t) = \prod_{t_{(j)} < t} (1 - \hat{\theta}_j) \quad \text{donde} \quad \hat{\theta}_j = \frac{d_j}{n_j}$$

Now, let us suppose that the sample is subject to right-censoring and the number of censored spells in the interval $[t_{(j)},t_{(j+1)})$ is $w_j$. In this case it is necessary to make some modifications since the number of individuals who leave the initial state is not known precisely. In this case, both the estimation of the risk function and the survival function are maintained. The only change is the composition of the risk function. Now the risk set at $t_{(j)}$ must be formed by the number of individuals who survive and are not censored just before $t_{(j)}$. The estimation of the survival function must not change under the censoring time since in those periods there must not be movement of the individuals to the final state. Nevertheless, the risk set and therefore the size of the steps of $\hat{S}(t)$ must change.

In the case where a censored time is registered equal to a lifetime $t_{(j)}$ the idea of adjusting the right the censored time in an infinitesimal amount greater than $t_{(j)}$ is adopted. In this way an individual with censored time equal to $t_{(j)}$ is included in the set of those $n_j$ individuals who are at risk in $t_{(j)}$. Therefore, in the case of right-censored $n_j$ would be:

$$n_j = \sum_{i=j}^{M} (d_i + w_i) = N - \sum_{j=1}^{i-1} (d_i + w_i)$$

Another issue which needs clarifying is what happens in those situations where the longer time observed is a censored time instead of a time of leaving. In this case the estimator P.L. would be defined until the last moment of leaving observed. As consequence this survival function would not reach the value zero.

Therefore the estimator for the survival function would be written

$$\hat{S}(t) = \prod_{t_{(j)} < t} \left(1 - \frac{d_j}{n_j} \right) = \prod_{t_{(j)} < t} \left(\frac{n_j - d_j}{n_j} \right)$$
This estimator of the survival function is known in the literature as the limit product or the Kaplan Meier estimator.

We can derive an estimator for the variance of the estimator P.L., by the delta method. To this end we suppose that the $\hat{\theta}_j$ is approximately independent from variance given by the binomial expression $\text{Var}(\hat{\theta}_j) = \theta_j(1-\theta_j)/n_j$ when $n_j$ is large. Therefore an estimator of the variance of the survival function at $t$ is:

$$\text{Var}(\hat{S}(t)) = \left(\hat{S}(t)\right)^2 \sum_{t_{ij} < t} \frac{\hat{\theta}_j}{(1-\hat{\theta}_j)n_j}$$

This expression is known as the Greenwood.

According to Kalbfleisch and Prentice (1980), with $T$ continuous and smooth conditions of censoring it can be shown that the estimator PL is asymptotically a Gaussian process, in such a way an interval of confidence of 95% can be calculated for the survival function at any $t$ given by $\hat{S}(t) \pm 1.96 \left(\hat{\nu}(\hat{S}(t))\right)^{1/2}$. However these confidence intervals can contain sometimes impossible values, outside the range $[0,1]$. To avoid this the authors propose applying the normal asymptotic distribution to some transformation of $S(t)$ so that the range is not restricted, based on the function:

$$\hat{\nu}(t) = \ln(-\ln\hat{S}(t))$$
Bibliografía


