Modeling Spatial Autocorrelation in Spatial Interaction Data:
A Comparison of Spatial Econometric and Spatial Filtering Specifications

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Abstract. Spatial interaction models of the gravity type are widely used to model origin-destination flows. They draw attention to three types of variables to explain variation in spatial interactions across geographic space: Variables that characterise an origin region of a flow, variables that characterise a destination region of a flow, and finally variables that measure the separation between origin and destination regions. These models are said to be misspecified if their residuals are spatially autocorrelated, violating the independence assumption. This paper outlines and compares two approaches, the spatial econometric and the eigenfunction-based spatial filtering approach, to deal with the issue of spatial autocorrelation among flow residuals. An example using patent citation data that capture knowledge flows across 112 European regions serves to illustrate the application and the comparison of the two approaches.

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1 Introduction

Spatial autocorrelation in geocoded data can be a serious problem, rendering conventional statistical analysis unsound and requiring specialised spatial analytical tools. Spatial autocorrelation refers to the pairwise correlation of georeferenced observations for a single variable. Correlation retains its classical meaning of association, whereas 'auto' means self and spatial describes the manner in which self-correlation arises. Autocorrelation is attributable to the configurational arrangement of observations. The problem arises in situations where the observations are nonindependent over space. That is, nearby spatial units (regions) are associated in some way. Sometimes, this association is due to a poor match between the spatial extent of a phenomenon of interest and the administrative units for which data are available. Sometimes, it is due to a spatial spillover effect. The complications are similar to those found in time series analysis, but are exacerbated by the multidirectional, two-dimensional nature of dependence in space rather than the unidirectional, one-dimensional nature in time.

Spatial interaction or flow data pertain to measurements each of which is associated with a link or a pair of origin-destination locations that represent points or areas (regions) in space. While a voluminous literature exists for spatial autocorrelation with a focus of interest on the specification and estimation of models for cross-sectional attribute data, there is scant attention paid to its counterpart in spatial interaction data. For example, there is no explicit reference to spatial flows data in some of the commonly cited spatial econometric and statistics texts, such as Anselin (1988) and Cressie (1991). But Griffith (1988, pp. 66-79) implicitly addresses this topic, and Griffith and Jones (1980) treat this very problem. Furthermore, some relevant research has been done about network autocorrelation (see Black 1992, Black and Thomas 1998, Tiefelsdorf and Braun 1999); but this work treats flows in an indirect way.

Modelling spatial interactions has a long and distinguished history in geography and regional science (see, especially, Wilson 1970, Sen and Smith 1995). Spatial interaction models focus on dyads of regions rather than on individual regions. They aim to explain variation of spatial interaction across geographic space. In doing so, they draw attention to three types of push-pull variables: those relating to properties of the origin regions (origin factor); those relating to properties of the destination region (destination factor); and, those relating to the spatial separation between origin and destination regions (separation factor). Spatial interaction models are said to be misspecified if the residuals are spatially autocorrelated, violating the independence assumption. This problem has been largely neglected so far, with very few exceptions [see, for example, Brandsma and Ketellapper 1979, Griffith and Jones 1980, Bolduc et al. 1992, 1995, LeSage and Pace 2005, Fischer et al. 2006a]. This neglect may be because spatial interaction models are more complex than models for the geographic distribution of attribute data, with each region being associated with several values as an origin as well as a destination so that specification of the autocorrelation structure is less obvious.

This paper outlines and compares two approaches that might be used to account for spatial autocorrelation in a spatial interaction modelling context. One approach involves directly modelling spatial autocorrelation among flow residuals by introducing a spatial error structure that reflects origin and/or destination autoregressive dependence among origin-destination flows. This view leads to spatial autoregressive model specifications that represent not only extensions of the conventional spatial interaction models, but also extensions of the spatial regression models, the workhorses of applied spatial econometrics.

The other approach, eigenfunction spatial filtering, starts from the misspecification interpretation perspective of spatial autocorrelation, which assumes that spatial autocorrelation among flow residuals is induced by missing origin and destination variables, which themselves are spatially autocorrelated. The approach itself is a non-parametric technique that removes the inherent spatial autocorrelation from spatial interaction models by introducing appropriate synthetic surrogate variates (i.e., spatial filters) for the origin and destination variables, and exploiting hereby an eigenfunction decomposition associated with Moran's *I* (*MI*) statistic of spatial autocorrelation.

The structure of the paper is as follows. The section that follows sets forth the context and framework for the discussion, with a particular focus on the log-additive spatial interaction model version, one of the most common specifications employed in applied spatial interaction data analysis, as well as the Poisson regression generalised linear model version, today's preferred specification. Section 3 outlines the spatial econometric approach that generalises the classical spatial interaction models to spatial econometric origin-destination flow models. These models are formally equivalent to conventional spatial regression models. But they differ in terms of the data analysed and the way in which the spatial weights matrix is defined. Section 4 moves attention to the eigenfunction-based spatial filtering approach that accounts for the inherent spatial autocorrelation from spatial interaction models with a composite map pattern component (i.e., a spatial filter), rather than simply identifying a global spatial autocorrelation parameter for a spatial autoregressive process. The aim of this non-parametric approach is to control spatial autocorrelation by introducing appropriate synthetic variables that serve as surrogates for spatially autocorrelated missing origin and destination variables. This shift in focus leads to spatial filter variants of the classical spatial interaction model.

Patent citation data that capture knowledge flows across 112 European regions are used in Section 5 to compare the workings of the two approaches. The final section concludes the paper with a final comparison of the two approaches.

2 Background

Suppose we have a spatial system consisting of n regions. Let m_{ij} (i, j=1, …, n) denote observations on random variables, say M_{ij}, each of which corresponds to a movement of people, information or communication from region i to region j. The M_{ij} are assumed to be independent random variables. They are sampled from a specified probability distribution dependent upon some mean, say μ_{ij}. Let us assume that no a priori information is given about the row and column totals of the flow matrix $[m_{ij}]$. Then the expected flows $\mu_{ij} = E[M_{ij}]$ between origin i and destination j may be modelled by

$$\mu_{ij} = c\ a_i^{\alpha}\ b_j^{\beta}\ f_{ij}(\theta\, d_{ij}) \qquad (1)$$

with

$$f_{ij} = \exp\left[\sum_{k=1}^{K} \theta_k\ d_{ij}^{(k)}\right] \qquad (2)$$

and with $d_{ij}^{(k)} = (d_{ij}^{(1)}, ..., d_{ij}^{(K)})$ and $\theta = (\theta_1, ..., \theta_K)$, where c is a constant of proportionality, and α, β and θ indicate the relative importance of the origin, destination and separation variables. The singly indexed variables a_i and b_j characterise the origin and destination of interaction, respectively. These can be the number of opportunities in either the origins or the destinations. The doubly indexed variable d_{ij} is a multivariate measure of spatial separation that jointly varies across all origin and destination combinations, and represents the deterring influence of the separation between origins and destinations. On the one hand, the larger the number of opportunities becomes at an origin or destination, the larger the expected interaction flow will be; on the other hand, the larger the separation becomes between i and j, the smaller the interaction flow is expected to be. The origin specific parameter α and the destination specific parameter β as well as the cross-regional parameter θ and the overall baseline parameter c are estimated in order to match the expected flows μ_{ij} as closely as possible with the observed flows m_{ij}.

The log-transformed model specification

For linear model analysis purposes, two implicit assumptions are that a, b and f are positive, and that the factors a and b are independent of f. Thus, model (1)-(2) can be expressed equivalently as a log-additive model[1] of the form

$$y(i,j) = \kappa + \alpha\, a(i) + \beta\, b(j) + \sum_{k=1}^{K} \theta_k \, \overset{(k)}{d}(i,j) \ + \varepsilon(i,j) \tag{3}$$

where $y(i,j) \equiv \ln(\mu_{ij})$, $\kappa \equiv \ln(c)$, $a(i) \equiv \ln(a_i)$, $b(j) \equiv \ln(b_j)$ and $d(i,j) \equiv d_{ij}$. Of note is that the back-transformation of this log-linear specification results in the error structure being multiplicative. Assume that $E(\varepsilon)=0$ and that $a(i)$ and $b(j)$ are measured without error [the standard minimal assumptions for least squares regressions]; then we obtain the ordinary least squares estimator (OLS), say $\tilde{\boldsymbol{\gamma}}$, for $\boldsymbol{\gamma} = (\kappa, \alpha, \beta, \theta_1 ..., \theta_K)^T$, as the solution to the matrix equation

$$(\boldsymbol{X}^T \boldsymbol{X})\, \tilde{\boldsymbol{\gamma}} = \boldsymbol{X}^T \boldsymbol{y} \tag{4}$$

with

$$\boldsymbol{y}^T = \left[y(1,1),\ ...,\ y(1,n),\ ...,\ y(n,1),\ ...,\ y(n,n) \right] \tag{5}$$

given that the (N, 1)-vector $\boldsymbol{d}^{(k)} = [\overset{(k)}{d}(1,1), ..., \overset{(k)}{d}(n,n)]^T$ is the vectorised form of the n-by-n separation matrix $[d_{ij}^{(k)}]$, $\boldsymbol{a} = [a(1),..., a(1),..., a(n),..., a(n)]^T$ and $\boldsymbol{b} = [b(1),..., b(n), ..., b(1),..., b(n)]^T$ are N-by-1 vectors, appropriately indexed over the $N = n^2$ values, and where $\boldsymbol{y}$ denotes the N-by-1 vector of observations on the interaction variable, $\boldsymbol{X}$ is the (N, K+3)-matrix of observations on the explanatory variables including the origin, destination, separation variables and an intercept. $\boldsymbol{\gamma}$ is the associated (K+3)-by-1 parameter vector. The N-by-1 vector $\boldsymbol{\varepsilon} = [\varepsilon(1,1), ..., \varepsilon(n,n)]^T$ denotes the vectorised form of $[\varepsilon_{ij}]$.

Depending upon the assumptions made about the variance-covariance matrix of $\boldsymbol{\varepsilon}$, the estimators derived from equation (4) may or may not be efficient. But equation (4) is an unbiased equation (Durbin 1960) in the sense that

[1] Note in some cases $y_{ij} = 0$ indicating the absence of flows from i to j. This leads to the so-called zero problem since the logarithm then is undefined. There are several pragmatic solutions to this problem, with adding a small constant to the zero elements of $[y_{ij}]$ being widely used.

$$E[\boldsymbol{X}^T \boldsymbol{X} \hat{\boldsymbol{\gamma}}] = \boldsymbol{X}^T \boldsymbol{X} \ E[\hat{\boldsymbol{\gamma}}] = \boldsymbol{X}^T \ E[\boldsymbol{y}] = \boldsymbol{X}^T \boldsymbol{X} \ \boldsymbol{\gamma} \tag{6}$$

where $E[.]$ denotes the expectation operator. From equation (6) we see that

$$E[\hat{\boldsymbol{\gamma}}] = \boldsymbol{\gamma} \tag{7}$$

provided that $(\boldsymbol{X}^T \boldsymbol{X})^{-1}$ exists. That is, the data must not be perfectly collinear. This result holds whatever dispersion matrix, $\sigma^2 \boldsymbol{V}$, is postulated for the disturbance, $\boldsymbol{\varepsilon}$.

A violation of the above assumptions may lead to two problems: (i) spatial autocorrelation among the $\boldsymbol{X}$-variables, and (ii) spatial autocorrelation among the residuals, $\boldsymbol{\varepsilon}$. Both problems may well arise, but neither implies the other. If (i) holds, this will affect the matrix $(\boldsymbol{X}^T \boldsymbol{X})^{-1}$, or $(\boldsymbol{X}^T \boldsymbol{V}^{-1} \boldsymbol{X})^{-1}$ in general, and thus the variance estimates of the coefficients. If (ii) holds, then the basic assumption of a scalar dispersion matrix for the disturbances, $\boldsymbol{\varepsilon}$, is violated; that is, $E\left[\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T\right] = \sigma^2 \boldsymbol{V}$ where $\boldsymbol{V} \neq \boldsymbol{I}$. Thus, there will be an extra $\boldsymbol{V}$ matrix in the expressions, and generalised rather than ordinary least squares should be used for estimation purposes. If $\boldsymbol{V}$ is unknown, as is generally the case, then some form of iterative generalised least squares should be performed. In this case the parameter estimates will be consistent, but not necessarily unbiased. But this is true for every regression problem (Cliff et al. 1974).

The Poisson model specification

Flowerdew and Aitkin (1982) argue that Equation (3) coupled with a normal probability model provides an incorrect specification for spatial interaction models. They specify the following alternative model:

$$\mathrm{P}\{y(i,j)\} = \frac{\exp(-\lambda(i,j))\,\mu(i,j)^{y(i,j)}}{y(i,j)!} \tag{8}$$

where $\mathrm{P}\{.\}$ denotes probability, and the expected value, $\mu(i,j)$, is given by equation (1). Equation (8) models flows between origin i and destination j as inter-point movement counts; hence the specification of a discrete distribution. Flowerdew and Aitkin show the superiority of Poisson regression over OLS log-linear regression based on equation (3). Later, Flowerdew and Lovett (1988) extend equation (8) to singly- and doubly-constrained spatial interaction models (see Wilson 1970), again assuming independent origin/destination factors. Unfortunately, this Poisson probability model formulation does not incorporate spatial dependencies in the origin and destination terms. Consequences of overlooking such spatial structure effects are conceptualized in Curry (1972), with their presence empirically demonstrated by Griffith and Jones (1980). One possible reformulation to account for them was posited by Bolduc et al. (1989, 1992, 1995), with a more recent discussion of this topic by Tiefelsdorf (2003).

3 The Spatial Econometric Perspective

The spatial econometric perspective involves capturing spatial dependence in the error term by means of spatial weight structures that model dependence between the *N* origin-destination pairs in a manner consistent with the conventional log-additive spatial interaction model given by equation (3). This perspective results in the following spatial econometric origin-destination flow model:

$$\boldsymbol{y} = \boldsymbol{X}\,\boldsymbol{\gamma} + \rho\,\boldsymbol{W}\,\boldsymbol{\varepsilon} + \boldsymbol{\eta}\ , \tag{9}$$

which is a log-additive spatial interaction model with a *N*-by-1 error vector $\boldsymbol{\varepsilon} = \rho\,\boldsymbol{W}\,\boldsymbol{\varepsilon} + \boldsymbol{\eta}$, where η is an uncorrelated – and homoscedastic – normal error term that satisfies the classical assumptions of independent and identically distributed with zero mean and constant variance σ^2, $\boldsymbol{W}$ is a row-standardized spatial weights matrix that represents an *N*-by-*N* non-negative, sparse matrix, and ρ is a scalar parameter that reflects the nature and degree of spatial dependence and which is typically referred to as the spatial autoregressive parameter. This latter matrix captures dependency relations among the observations that reflect flows from origins to destinations in the system of *n* regions,. Use of matrix $\boldsymbol{W}$ allows the restriction $|\,\rho\,|<1$ to be invoked; if $|\,\rho\,|>1,$ the models would be explosive and non-stationary.

A key issue here is how to construct a meaningful spatial weights matrix in the case where the *N*-by-*N* vector of observations reflects flows from all origins to all destinations. We follow Fischer et al. (2006a) to construct a weights matrix, ${}^{o}\boldsymbol{W} + {}^{d}\boldsymbol{W}$, that captures origin and destination spatial dependence. Origin spatial dependence refers to the tendency for a flow from origin region *i* to any destination region to be of similar magnitude to its neighbours. Destination spatial dependence conveys the idea that the flow to a destination region *j* from any origin region tends to involve flows of similar magnitudes to its neighbours (see LeSage and Pace 2005).

Specification of origin-based spatial dependence. To capture origin-based spatial dependence, we define

$$ {}^{o}w(i,j;\,r,s)=\begin{cases}1 \ \text{ if } \ j=s \text{ and } c_{ir}=1\\ 0 \ \text{ otherwise,}\end{cases} \tag{10} $$

where c_{ir} is the element of a conventional n-by-n first order contiguity matrix that defines whether or not the origin regions i and r are contiguous:

$$ c_{ir}=\begin{cases}1 \ \text{ if } \ i\neq r, \text{ and } i \text{ and } r \text{ have a common border}\\ 0 \ \text{ otherwise.}\end{cases} \tag{11} $$

This origin-based spatial weights matrix specifies an origin-based neighbourhood set for each origin-destination pair (i, j). An element ${}^{o}w(i,j;\,r,s)$ defines an origin-destination pair (r, s) as being a 'neighbour' of (i, j) if the origin regions i and r are contiguous spatial units and $j = s$. It is convenient to work with a row-standardised form of ${}^{o}\boldsymbol{W}$. In order to achieve this, each element of the matrix has to be divided by its respective row sum so that the row elements of the standardised matrix sum to one.

Specification of destination-based spatial dependence. Analogously, we define a row-standardised destination-based N-by-N spatial weights matrix ${}^{d}\boldsymbol{W}=\{{}^{d}w(i,j;\,r,s)\}$ in which we capture destination-based dependence as follows:

$$ {}^{d}w(i,j;\,r,s)=\begin{cases}1 \ \text{ if } \ i=r \text{ and } c_{js}=1\\ 0 \ \text{ otherwise,}\end{cases} \tag{12} $$

and

$$ c_{js}=\begin{cases}1 \ \text{ if } \ j\neq s, \text{ and } j \text{ and } s \text{ have a common border}\\ 0 \ \text{ otherwise.}\end{cases} \tag{13} $$

Maximum Likelihood estimation of the origin-destination flow model described above involves maximisation of the log-likelihood function [concentrated for $\boldsymbol{\gamma}$ and σ^2] with respect to the spatial autoregressive parameter ρ:

$$ \mathcal{L}(\rho)=\ln|\boldsymbol{I}-\rho\boldsymbol{W}|-N\ln\Big[(\boldsymbol{y}-\boldsymbol{X}\ \tilde{\boldsymbol{\gamma}}(\rho))^{T}\ \ (I-\rho\boldsymbol{W})^{T}(I-\rho\boldsymbol{W})\ \ (\boldsymbol{y}-\boldsymbol{X}\ \tilde{\boldsymbol{\gamma}}(\rho))\Big] \tag{14} $$

$$ \tilde{\boldsymbol{\gamma}}(\rho)=(\boldsymbol{X}^{T}\ (I-\rho\boldsymbol{W})^{T}(I-\rho\boldsymbol{W})\ \boldsymbol{X})^{-1}\boldsymbol{X}^{T}[(I-\rho\boldsymbol{W})^{T}(I-\rho\boldsymbol{W})]^{-1}\ \boldsymbol{y} \tag{15} $$

$$\tilde{\sigma}(\rho)^2 = \tfrac{1}{N}(\boldsymbol{y}-\boldsymbol{X}\ \tilde{\boldsymbol{\gamma}}(\rho))^T\ (I-\rho\boldsymbol{W})^T(I-\rho\boldsymbol{W})\ (\boldsymbol{y}-\boldsymbol{X}\ \tilde{\boldsymbol{\gamma}}(\rho)). \tag{16}$$

Equation (14) is maximised with respect to ρ where ρ is restricted to the interval $(-\lambda_{\min}^{-1}, \lambda_{\max}^{-1})$, the inverse of the extreme eigenvalues of the spatial weights matrix; $\lambda_{\min} < 0$ and $\lambda_{\max} = 1$ represent the minimum and maximum eigenvalues of $\boldsymbol{W}$. ML estimates of $\boldsymbol{\gamma}$ and σ^2 are found by substituting the optimal value of ρ into equations (15) and (16).

The major difficulty in numerical maximisation of the concentrated log-likelihood function is the necessity of evaluating the *N*-by-*N* log-determinant of $(\boldsymbol{I}-\rho\boldsymbol{W})$ at each step. The eigenvalue approach to computing this term suggested by Ord (1975) becomes computationally intensive even when *N* is small. But $\boldsymbol{W}$ is a large sparse *N*-by-*N* matrix by construction so that the use of sparse matrix Cholesky factorisation techniques means maximum likelihood estimation of these models is computationally feasible for larger spatial interaction systems. Execution time speed improvements come at no cost to statistical accuracy, as is illustrated in Fischer et al. (2006a).

Some simplification can be attained here because the *N*-by-*N* matrices can be rewritten in Kronecker product form. Consequently, the original set of *n* eigenvalues is repeated *n* times for the origin and *n* times for the destination spatial autoregressive terms. Griffith (e.g., 1992, 2004b) shows how the Jacobian term based upon these eigenvalues can be approximated, resulting in a dramatic simplification for computational purposes. One example of this type of decomposition is presented by Griffith (1996b) for space-time data.

4 The Eigenfunction Spatial Filtering Approach

The eigenfunction spatial filtering approach represents an alternative methodology to account for spatial autocorrelation in a spatial interaction model. The primary motivation for this approach in the current context is to allow spatial analysts to compute OLS estimators for the parameters of the log-additive spatial interaction model, as well as generalised linear model Poisson regression spatial interaction parameter estimates, while ensuring that the required model assumptions are met. The approach outlined in this section derives from the eigenfunction spatial filtering approach devised by Griffith (1996a, 2000, 2002, 2003, 2004a) for attribute data. This approach is non-parametric in nature and aims to control for spatial autocorrelation by introducing appropriate synthetic variables that serve as surrogates for spatially autocorrelated missing origin and destination variables. These synthetic variables are derived as linear combinations of eigenvectors of the following modified version of the conventional n-by-n binary 0-1 contiguity matrix $\boldsymbol{C}$:

$$(\boldsymbol{I}-\boldsymbol{1}\boldsymbol{1}^T\tfrac{1}{n})\ \boldsymbol{C}\ (\boldsymbol{I}-\boldsymbol{1}\boldsymbol{1}^T\tfrac{1}{n})\,, \tag{17}$$

where $\boldsymbol{I}$ is the n-by-n identity matrix, and $\boldsymbol{1}$ an n-by-1 vector of ones. This particular matrix expression appears in the numerator of Moran's I (*MI*) statistic of spatial autocorrelation defined for attribute data. Tiefelsdorf and Boots (1995) show that all of the eigenvalues of expression (17) relate to distinct *MI* values.

An eigenfunction linked to some geographic contiguity matrix $\boldsymbol{C}$ may be interpreted in the context of latent map pattern as follows (Getis and Griffith 2002): The first eigenvector, say $\boldsymbol{E}_1$, is the set of numerical values that has the largest *MI* value achievable for any set of numerical values, for the given geographic contiguity matrix. The second eigenvector, $\boldsymbol{E}_2$, is the set of numerical values that has the largest achievable *MI* for any set of numerical values that is uncorrelated with $\boldsymbol{E}_1$. This sequential construction of eigenvectors continues through $\boldsymbol{E}_n$, which is the set of numerical values that has the largest negative *MI* achievable by any set of numerical values which is uncorrelated with the preceding (n-1) eigenvectors. These n eigenvectors describe the full range of all possible mutually orthogonal and uncorrelated map patterns, and may be interpreted as synthetic map variables that represent specific natures (that is, positive or negative) and degrees (that is, negligible, weak, moderate, strong) of potential spatial autocorrelation.

The eigenvector spatial filtering approach, based upon a stepwise selection criterion, adds a minimally sufficient set of eigenvectors as proxies for missing origin and destination variables, and in doing so eliminates spatial autocorrelation among the observations by inducing mutual dyad error independence. This leads to a spatial filter specification of the spatial interaction model (1)-(2) that may be described as

$$\mu_{ij} = c\ \exp[\sum_{q=1}^{Q} E_{iq}\,\psi_q]\ a_i^{\alpha}\ \ \exp[\sum_{r=1}^{R} E_{jr}\,\varphi_r]\ b_j^{\beta}\ \ \exp[\sum_{k=1}^{K}\theta_k\ \overset{(k)}{d}(i,j)] \tag{18}$$

where μ_{ij}, $\overset{(k)}{d}(i,j)$, c, α, β, θ_k (k=1, …, K), a_i and b_j are defined as above, Q and R denote selected subsets of the n eigenvectors that have been chosen by supervised selection to furnish a good description of the original origin and destination variables, respectively, and ψ_q and φ_r are the respective coefficients for the linear combinations of eigenvectors that constitute the origin and destination spatial filters, namely $\sum_{q=1}^{Q} E_{iq}\,\psi_q$ and $\sum_{r=1}^{R} E_{jr}\,\varphi_r$. For these spatial filters, which are linear combinations of the eigenvectors of expression (17) and represent the spatial autocorrelation components of the missing origin and destination variables, ψ_q $(q = 1,...,Q)$ and φ_r $(r = 1,...,R)$ are regression coefficients that indicate the relative importance of each distinct map pattern in accounting for spatial autocorrelation in the flows structure.

The spatial filtering model specification of the log-normal additive model

Spatial filter spatial interaction model (18) can be expressed equivalently in log-additive form, in order to link it to a normal probability distribution for the error term, as

$$y(i,j) = \kappa + \sum_{q=1}^{Q} E_{iq}\,\psi_q + \alpha\ a(i) + \sum_{r=1}^{R} E_{ir}\,\varphi_r + \beta\ b(j) + \sum_{k=1}^{K} \theta_k\, d^{(k)}(i,j) + \varepsilon(i,j)\ . \qquad (19)$$

OLS can be employed to estimate the model parameters. All conventional diagnostic statistics developed for linear regression analysis can be computed and interpreted without having to develop spatially adjusted counterparts. The major numerical difficulty of the spatial filter model version is that eigenfunctions have to be calculated, a formidable computational task for larger spatial interaction systems (i.e., large *n*). But this drawback is no more severe than computing the eigenvalues to calculate an n^2-by-n^2 matrix determinant.

Specification of a conventional Poisson spatial interaction model. Following the data organisation convention of Lesage and Pace (2005), an *n*-by-*n* spatial interaction matrix can be unfolded and have covariates attached to dyad flows data as follows:

Dyad label	ID_{origin}	$ID_{destination}$	Flow	Origin covariates	Destination covariates	Distance(origin, destination)
1	1	1	$y(1, 1)$	$\boldsymbol{x}_1$	$\boldsymbol{x}_1$	$d(1, 1)$
2	2	1	$y(2, 1)$	$\boldsymbol{x}_2$	$\boldsymbol{x}_1$	$d(2, 1)$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	n	1	$y(n, 1)$	$\boldsymbol{x}_n$	$\boldsymbol{x}_1$	$d(n, 1)$
n+1	1	2	$y(1, 2)$	$\boldsymbol{x}_1$	$\boldsymbol{x}_2$	$d(1, 2)$
n+2	2	2	$y(2, 2)$	$\boldsymbol{x}_2$	$\boldsymbol{x}_2$	$d(2, 2)$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
2n	n	2	$y(n, 2)$	$\boldsymbol{x}_n$	$\boldsymbol{x}_2$	$d(n, 2)$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
(n-1)n	1	n	$y(1, n)$	$\boldsymbol{x}_1$	$\boldsymbol{x}_n$	$d(1, n)$
(n-1)n+1	2	n	$y(2, n)$	$\boldsymbol{x}_2$	$\boldsymbol{x}_n$	$d(2, n)$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
n^2	n	n	$y(n, n)$	$\boldsymbol{x}_n$	$\boldsymbol{x}_n$	$d(n, n)$

The lower-case xs are 1-by-p vectors containing p covariates. The "flow" and "distance" column entries are extracted directly from n-by-n matrices. The "origin covariates" column entries are obtained from an n-by-p matrix $\boldsymbol{X}$ of covariates as follows: $\boldsymbol{1} \otimes \boldsymbol{X}$, where $\boldsymbol{1}$ is an n-by-1 vector of ones, and $\otimes$ denotes Kronecker product. The "destination covariates" column entries are obtained as follows: $\boldsymbol{X} \otimes \boldsymbol{1}$. Equation (3) as mean response and hence , and hence without the error term, can be estimated with this data organization via Poisson regression through the use of a generalised linear model algorithm coupled with a Poisson distribution link function. Intraregional unit flows can be eliminated by removing the n cases for which the origin and destination ids are the same (i.e., $\text{ID}_{\text{origin}} = \text{ID}_{\text{destination}}$). Parameter estimation can be achieved either with iteratively reweighted least squares or maximum likelihood techniques.

Specification of a spatial filter spatial interaction model. Spatial filter counterparts to the spatial econometric specification can be obtained in one of two ways: (i) by augmenting the set of covariates with the set of candidate eigenvectors – relating to equation (19); and, (ii) by estimating parameters for this augmented set with a Poisson regression – relating directly to equation (18). The origin candidate eigenvectors are obtained from $\boldsymbol{1} \otimes \boldsymbol{E}_K$, whereas the destination candidate eigenvectors are obtained from $\boldsymbol{E}_K \otimes \boldsymbol{1}$, where $\boldsymbol{E}_K$ is the set of candidate eigenvectors (e.g., those whose associated Moran Coefficient, when divided by the maximum possible Moran Coefficient, exceeds 0.25).

5 An Illustrative Application of the Approaches

Patent citation data are used to illustrate the way the two approaches might be applied to control for spatial autocorrelation among the residuals in a spatial interaction model. Such data recorded in patent documents are widely recognised as a rich and fruitful source for the study of the spatial dimension of innovations and technological change (see, for example, Jaffe and Trajtenberg 2002, Fischer et al. 2006b).

The Context

We use interregional patent citation flows as the dependent variable in the models. The data specifically relate to citations between European high-tech patents. By European patents we mean patent applications at the European Patent Office assigned to high-tech firms located in

Europe. High-technology is defined to include the ISIC-sectors of aerospace (ISIC 3845), electronics-telecommunication (ISIC 3832), computers and office equipment (ISIC 3825) and pharmaceuticals (ISIC 3522). Self-citations (i.e., citations from patents assigned to the same firm) have been excluded, given our interest in pure externalities as evidenced by *interfirm* knowledge spillovers.

It is well known that the observation of citations is subject to a truncation bias, because we observe citations for only a portion of the *life* of an invention. To avoid this bias in the analysis we have established a five-year window (e.g., 1985-1989, 1986-1990, 1987-1991) to count citations to a patent[2]. The observation period is 1985-1997 with respect to cited patents and 1990-2002 with respect to citing patents. The sample used in this contribution is restricted to inventors located in n = 112 regions, generally NUTS-2 regions, covering the core of 'Old Europe,' including Germany (38 regions), France (21 regions), Italy (20 regions), the Benelux countries (24 regions), Austria (8 regions) and Switzerland (one region), resulting in N = 12,432 interregional flows[3].

Subject to caveats relative to the relationship between citations and spillovers, these data allow us to identify and measure spatial separation effects for interregional knowledge spillovers in this interaction system of 112 regions. Our interest is focused on K = 3 measures: $\boldsymbol{d}^{(1)}$ is a N-by-1 vector that represents geographic distance measured in terms of the great circle distance [in km] between the regions represented by their economic centres; $\boldsymbol{d}^{(2)}$ is a N-by-1 country dummy variable vector that represents border effects measured in terms of the existence of country borders between the regions; and, technological proximity $\boldsymbol{d}^{(3)}$.

As we consider the distance effect on interregional patent citations, it is important to control for technological proximity between regions, as geographical distance could be just proxying for technological proximity. To do this we use a technological proximity index s_{ij} that defines the proximity between regions i and j in technology space. We divide the high-technology patents into 55 technological subclasses following the International Patent Code classification system. Each region is assigned a (55, 1)-technology vector that measures the share of patenting in each of the technological subclasses for a region. The technological proximity index s_{ij} between regions i and j is given by the uncentred correlation of their

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[2] For details on data construction see Fischer et al. (2006b). To obtain citations by any one patent application in year t, one needs to search the references made by all patent applications after year t. This is called the inversion problem that arises because the original data on citations come in the form of citations made, whereas we need dyads of cited and citing patents to construct interregional patent citations flows.

[3] Note that intraregional flows are left out of consideration. In the case of cross-regional inventor teams, the procedure of multiple full counting has been applied.

technological vectors. Two regions that patent exactly in the same proportion in each subclass have an index equal to one, while two regions patenting only in different subclasses have an index equal to zero. This index is appealing because it allows for a continuous measure of technological distance by the transformation $d_{ij} = 1 - s_{ij}$. Appropriate ordering leads to the *N*-by-1 vector $\boldsymbol{d}^{(3)}$.

The product $a_i\, b_j$ in equation (1) may be interpreted simply as the number of distinct (i, j)-interactions that are possible. Thus, it is reasonable to measure the origin factor in terms of the number of patents in the knowledge producing region *i* in the time period 1985-1997, and the destination factor in terms of the number of patents in the knowledge absorbing region *j* in the time period 1990-2002 to produce the *N*-by-1 vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

Application of the Spatial Autoregression Approach

This section reports the ML estimates of the spatial autoregressive model specification that reflects origin-destination spatial dependence of flows. We used the *spdep package* running on a Sun Fire V250 with 1.28 GHz and 8 GB RAM to create the spatial weights matrix from polygon contiguities, and the *errorsarlm* procedure based on Ng and Peyton's (1993) sparse matrix Cholesky algorithm to generate the ML estimates for the model. Using this algorithm, computation of the maximum likelihood estimates of the spatial econometric model required only 836 seconds, a remarkably short time considering that each iteration required calculating the determinant of a 12,432-by-12,432 matrix. Using the aforementioned Jacobian approximation approach could dramatically reduce this computation time.

Table 1 contains the parameter estimates of the model specification and its associated log-likelihood function value, together with those of the conventional log-additive spatial interaction model. Moving from the conventional spatial interaction model to the spatial econometric flow model reflecting spatial dependence at the origins and destinations increases the log-likelihood from -21,024.13 to -20,212.013. This is to be expected given the significance of the spatial autoregressive parameter that points to origin-destination spatial dependence ($\rho = 0.613$). It is clear that least squares, which ignores spatial dependence and assumes residual flows to be independent, produces a much lower likelihood function value. Capturing the dependences greatly reduces the residual variance and strengthens the inferential basis affiliated with the models.

Table 1. Log-normal additive spatial interaction models: The conventional model, the spatial autoregressive model specification using ${}^{o}\boldsymbol{W} + {}^{d}\boldsymbol{W}$, and the spatial filter model specification with 20 origin and 21 destination spatial filters

	The Conventional Log-Normal Additive Model [OLS]			The Spatial Autoregressive Model [ML]			The Spatial Filter Model Specification [OLS]		
	Estimates (Standard Error)	95% Confidence Limits		Estimates (Standard Error)	95% Confidence Limits		Estimates (Standard Error)	95% Confidence Limits	
Constant	-4.851 (0.236)	-5.315	-4.388	-4.658 (0.320)	-5.414	-4.532	-4.045 (0.249)	-4.533	-3.558
Origin Variable	0.594 (0.007)	0.580	0.608	0.593 (0.009)	0.576	0.610	0.587 (0.008)	0.571	0.603
Destination Variable	0.562 (0.007)	0.548	0.576	0.553 (0.009)	0.536	0.570	0.551 (0.008)	0.534	0.567
Geographical Distance	-0.181 (0.020)	-0.220	-0.142	-0.224 (0.038)	-0.296	-0.152	-0.238 (0.023)	-0.283	-0.193
Country Border	-0.592 (0.034)	-0.658	-0.526	-0.651 (0.054)	-0.754	-0.548	-0.671 (0.036)	-0.742	-0.600
Technological Distance	-2.364 (0.203)	-2.763	-1.966	-2.183 (0.212)	-2.586	-1.780	-2.638 (0.206)	-3.041	-2.235
Spatial Autoregressive Parameter	–	–	–	0.613 (0.011)	0.592	0.634	–		
Origin Spatial Filter	–	–	–	–	–	–	1 (0.374)	0.626	1.374
Destination Spatial Filter	–	–	–	–	–	–	1 (0.258)	0.742	1.258
Sigma Square	1.724			1.442			1.614		
Pseudo-R^2	0.563			0.597			0.719		
Log-likelihood	-21,024.128			-20,212.013			-20,591.301		
Moran's *I* (*p*-value)	0.193 (0.000)			-0.006 (0.939)			0.145 (0.000)		
Likelihood Ratio Test (*p*-value)	–			1,624.23 (0.000)			–		

But some important differences arise in the estimates and inferences that we would draw from the spatial econometric origin-destination flow model and the conventional spatial interaction model that ignores spatial dependence and assumes residual flows to be independent. Maximum likelihood, for example, ascribes a greater negative influence to geographical distance and national borders in creating friction that inhibits knowledge flows. Another difference in inferences pertains to the role of technological distance. The maximum likelihood estimate from the model specification reflects spatial dependence at origin and destination locations indicates a slightly less important negative influence.

Application of the Eigenfunction Spatial Filtering Approach

This section reports the OLS estimates of the spatial filter model specification of the log-normal additive model (see Table 1), and the ML estimates of the conventional Poisson model and its spatial filter model specification that reflects origin-destination spatial dependence of flows. A separate spatial filter is constructed for the origins and for the destinations. The 27 candidate eigenvectors (those, out of a total of 112, whose Moran Coefficient divided by the maximum Moran Coefficient is at least 0.25) were computed with a FORTRAN program using IMSL routines, and Poisson regression was executed with the SAS PROC GENMOD procedure.

Table 2. Poisson spatial interaction models: The conventional model and the spatial filter model specification with 23 origin and 16 destination spatial filters

	The Conventional Poisson Model [ML]			The Spatial Filter Model Specification [ML]		
	Estimates (Standard Error)	95% Confidence Limits		Estimates (Standard Error)	95% Confidence Limits	
Constant	-8.983 (0.112)	-9.202	-8.763	-7.428 (0.125)	-7.674	-7.183
Origin Variable	0.857 (0.006)	0.846	0.846	0.817 (0.007)	0.803	0.831
Destination Variable	0.835 (0.005)	0.825	0.846	0.783 (0.006)	0.771	0.795
Geographical Distance	-0.258 (0.012)	-0.387	-0.341	-0.583 (0.019)	-0.619	-0.546
Country Border	-0.364 (0.017)	-0.291	-0.225	-0.330 (0.012)	-0.353	-0.307
Technological Distance	-0.584 (0.064)	-0.709	-0.458	-1.553 (0.077)	-1.703	-1.402
Scale	1.508			1.356		
Origin Spatial Filter	–	–	–	1 (0.412)	0.588	1.412
Destination Spatial Filter	–	–	–	1 (0.202)	0.798	1.202
Sigma Square	57.736			34.855		
Pseudo-R^2	0.764			0.858		
Log-likelihood	40,919.915			51,973.184		
Moran's *I* (*p*-value)	0.163 (0.000)			0.094 (0.000)		
Likelihood Ratio Test (*p*-value)						

Table 3. Eigenvectors used to construct the origin and the destination spatial filters

Eigenvector	Moran Coefficient	Log-Normal Approximation Origin	Log-Normal Approximation Destination	Poisson Approximation Origin	Poisson Approximation Destination
$\boldsymbol{E}_1$	1.11180	1.42741	0.93952	1.6058	1.1994
$\boldsymbol{E}_2$	1.08506	0.81432	0.46405	1.5152	0.6819
$\boldsymbol{E}_3$	0.99199	**0**	**0**	-0.6757	**0**
$\boldsymbol{E}_4$	0.98400	-0.55321	-0.46187	-0.4873	**0**
$\boldsymbol{E}_5$	0.93913	-0.74278	-0.32688	-0.5387	**0**
$\boldsymbol{E}_6$	0.88555	**0**	**0**	0.5759	0.3286
$\boldsymbol{E}_7$	0.85454	0.48859	0.29019	0.8276	0.2586
$\boldsymbol{E}_8$	0.81034	1.04167	0.64129	0.9957	**0**
$\boldsymbol{E}_9$	0.78716	0.44417	**0**	0.8188	-0.3755
$\boldsymbol{E}_{10}$	0.74424	0.75294	0.28025	1.9399	0.9524
$\boldsymbol{E}_{11}$	0.67839	-0.41013	-0.47184	-0.2653	-0.6082
$\boldsymbol{E}_{12}$	0.65070	**0**	0.21490	-0.3584	-0.3663
$\mathbf{E}_{13}$	0.62828	0.55030	0.30766	**0**	**0**
$\boldsymbol{E}_{14}$	0.61328	-0.37897	-0.61055	**0**	-0.5046
$\boldsymbol{E}_{15}$	0.56651	0.50432	0.50477	**0**	**0**
$\boldsymbol{E}_{16}$	0.53836	-0.41588	0.36152	-0.3242	0.2610
$\boldsymbol{E}_{17}$	0.51970	-0.37527	**0**	-0.2258	**0**
$\boldsymbol{E}_{18}$	0.51434	0.51669	0.28619	0.3962	-0.2542
$\boldsymbol{E}_{19}$	0.48048	0.91238	0.76615	0.7568	0.4077
$\boldsymbol{E}_{20}$	0.44250	-0.34447	**0**	-0.4618	**0**
$\boldsymbol{E}_{21}$	0.42450	**0**	0.20912	0.6667	0.5700
$\boldsymbol{E}_{22}$	0.39132	**0**	-0.36015	-0.7781	-0.7835
$\boldsymbol{E}_{23}$	0.35404	0.43412	**0**	1.1175	0.6778
$\boldsymbol{E}_{24}$	0.34998	-0.56182	-0.29573	**0**	**0**
$\boldsymbol{E}_{25}$	0.32231	-0.78679	-0.43436	-0.8296	**0**
$\boldsymbol{E}_{26}$	0.29816	**0**	-0.28623	0.3795	0.2891
$\boldsymbol{E}_{27}$	0.28422	**0**	-0.24553	0.3501	**0**

The selected eigenvectors together with their estimated coefficients and associated levels of spatial autocorrelation, are summarized in Table 3. The origin and destination spatial filters for the log-normal additive model respectively contain 20 and 21 eigenvectors, and capture moderate positive spatial autocorrelation contained in the conventional spatial interaction model residuals. Maps of these two spatial filters appear in Figures 1a and 1b. The origin and destination spatial filters for the Poisson model contain 23 and 16 eigenvectors respectively, and capture moderate positive spatial autocorrelation contained in the basic Poisson model residuals. Maps of these two spatial filters appear in Figures 1c and 1d. Pairwise relationships between these four spatial filters are portrayed in Figure 2.

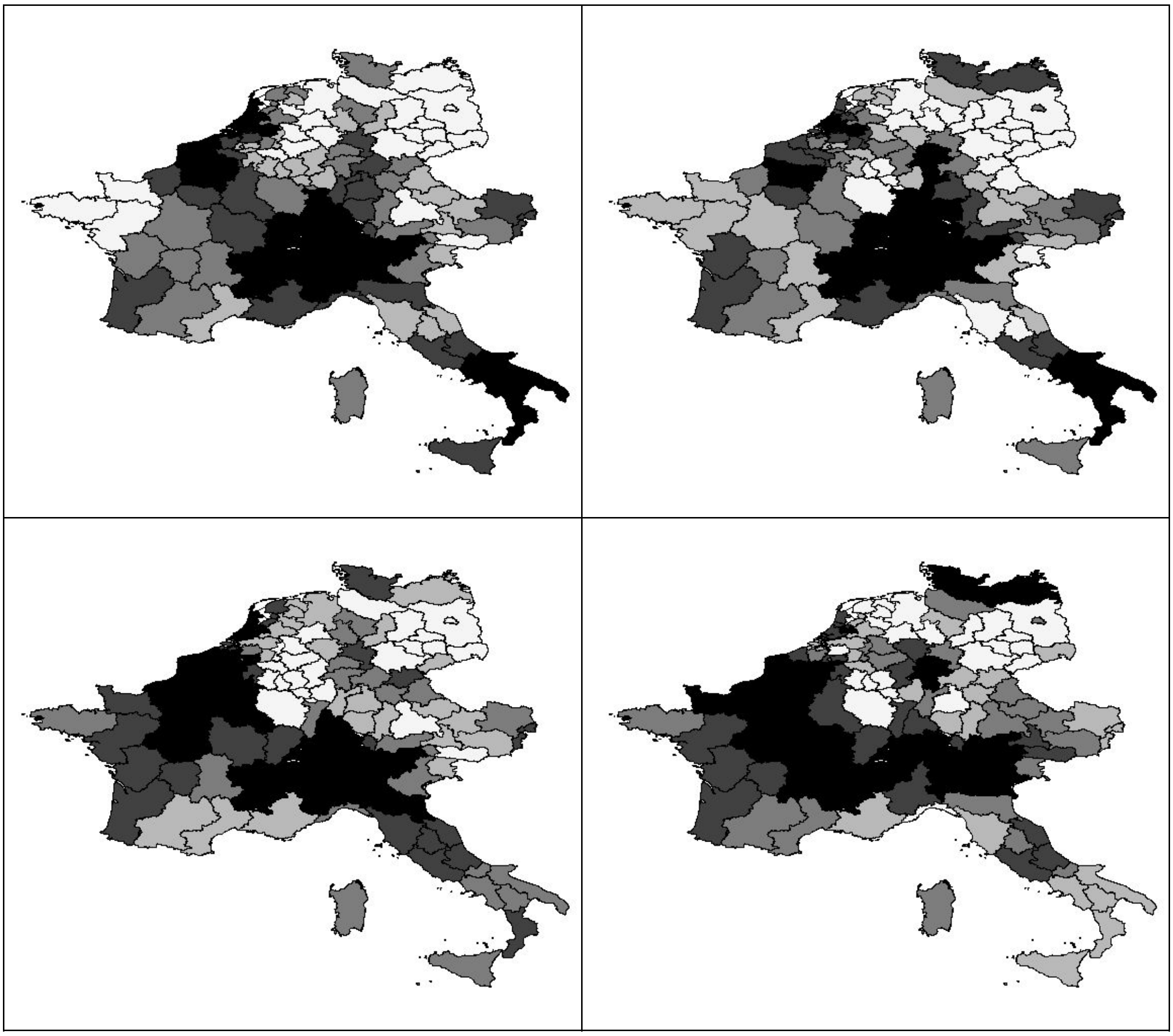

Figure 1. Spatial filters for the patent citation data, whose relative values are proportional to the darkness of the gray scale. (a) Top left: log-linear additive model origin spatial filter; Moran Coefficient = 0.778, Geary Ratio = 0.331. (b) Top right: log-linear additive model destination spatial filter; Moran Coefficient = 0.731, Geary Ratio = 0.349. (c) Bottom left: Poisson model origin spatial filter; Moran Coefficient = 0.781, Geary Ratio = 0.328. (d) Bottom right: Poisson model destination spatial filter; Moran Coefficient = 0.751, Geary Ratio = 0.370

Again some important differences arise in parameter estimates and inferences that we would draw from the spatial filter specification of the conventional log-normal additive spatial interaction model (see Table 1) as well as the spatial filter specification of the Poisson spatial interaction model (see Table 2). *First*, for the log-normal additive model, standard errors increase, as expected, but only slightly. In addition, accounting for spatial autocorrelation effects results in the importance of the origin and destination factors decreasing, while a greater negative influence is ascribed to geographical and technological distances as well as national borders in creating friction that inhibits knowledge flows. *Second,* Moran's *I* indicates that spatial autocorrelation among residuals is captured, but only modestly. *Third*, similar slight increases in standard errors occur with the Poisson specification. This respecification coupled with an accounting for spatial autocorrelation effects indicates that origin and destination factors play a more important role in generating spatial interaction than is revealed by the log-normal additive model, with geographic distance playing a much more important role, too, whereas the importance of technological distance and national borders appears to be overemphasized by the log-normal additive model.

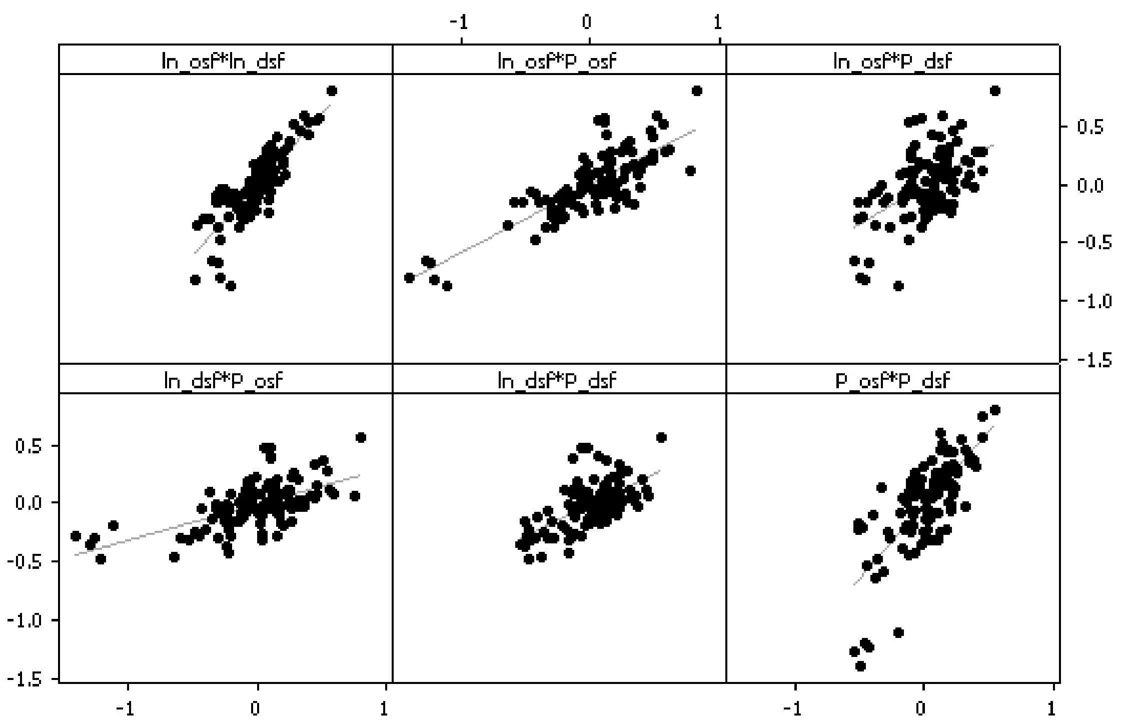

Figure 2. Scatterplots for spatial filter cross-correlations. The corresponding bivariate correlations range from 0.529 to 0.835

Considerable similarities exist between the map patterns captured by each of the four spatial filters. The bivariate correlation between the two origin spatial filters is 0.803 (see Figure 2), both highlighting a Switzerland-southern France-northern Italy focal region. This focus is less conspicuous with the two destination spatial filters, whose correlation is only 0.593. Both pairs of spatial filters suggest that much of the northern part of continental Europe forms a cluster, too. But the log-normal additive model noticeably differs from the Poisson model in terms of southern Italy, for both origin and destination spatial filters.

6 Summary and Conclusions

Two effective approaches to account for spatial autocorrelation in a spatial interaction model are described and demonstrated. Both approaches give researchers tools that aid in the proper specification of spatial interaction models. These approaches seem to yield similar results, but are somewhat different in the perspective within which each views the problem of spatial autocorrelation.

The spatial econometric approach is derivative of the literature on spatial autocorrelation in a cross-sectional spatial regression context. As such, it expresses spatial autocorrelation through the specification of a spatial stochastic process. But while the notion of spatial autocorrelation in a conventional spatial regression context involving a sample of n regions relies on a n-by-n spatial weights (connectivity) matrix, the notion of spatial autocorrelation in a spatial interaction context relies on a N-by-N (i.e., n^2-by-n^2) spatial weights matrix. The spatial weights matrix captures origin-based and destination-based dependency relations among the observations that influence flows from origins to destinations in a system of n regions. The resulting spatial econometric origin-destination flow models are formally equivalent to conventional spatial regression models, but differ in terms of the data analysed and the manner in which the spatial weights matrix is defined. Although one drawback of this approach is the computation of an N-by-N matrix determinant, the use of sparse matrix techniques puts maximum likelihood estimation of these models within the computational reach for larger spatial interaction systems.

Eigenvector spatial filtering furnishes an alternative methodology that enables to capture spatial autocorrelation effects within a spatial interaction model. This approach makes use of the misspecification interpretation of spatial autocorrelation and shifts attention from spatial autocorrelation in the dependent variable and, thus, the residuals, to spatial autocorrelation arising from origin and destination factors that is reflected in flows between pairs of these locations. In doing so, it allows for spatial interaction models where the desire is to avoid especially a log-linear spatial autoregressive specification coupled with a log-normally distributed error term, and to employ a generalized linear model formulation coupled with a Poisson distributed response variable. Although one drawback of this approach is the computer intensive eigenfunction calculations followed by time-consuming stepwise selection of eigenvectors, advances in computer technology continually increase the size of a geographic system for which these models are computationally feasible.

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