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A BAYESIAN APPROACH TO ANALYZE REGIONAL ELASTICITIES

ABSTRACT:

This paper presents the bayesian approach to analyze regional elasticity distributions with a regular translog cost function. It is known that a proper statistical analysis of elasticities can be performed only with the bayesian approach. We can take advantage of this methodology to form reasonable priors using national data. This way we can produce sounder inferences without much elicition by the analyst. To compare results this approach is applied to a cost function for the main regions in Italy with a diffuse prior too. Price and substitution elasticities are derived conditional on factor shares or covariates. The low posterior probability than inequality constraints hold with an noninformative prior shows how bayesian methods can be fruitfully employed to compare regional elasticities with a proper prior obtained from national data.

1. Introduction

Bayes analysis is straightforward, but it is still not often used in applied econometrics in regional science. I present a brief introduction to this approach[1]. Let θ a vector of parameters in which we are interested and y a vector of observations from a density $f(y/\theta)$ that is identical to the likelihood function $\ell(\theta/y)$ that contains all the sample information about the parameters. A priori analyst's knowledge about parameters is summarized by a subjective probability distribution $f(\theta)$. Therefore the joint distribution over parameters and observations is:

$$h(\theta,y) = f(\theta/y)f(y) = f(y/\theta)f(\theta)$$

that yields the Bayes theorem:

$$f(\theta/y) = \frac{f(y/\theta)f(\theta)}{f(y)} = \frac{\ell(\theta/y)f(\theta)}{f(y)} \propto \ell(\theta/y)f(\theta)$$

which states that the posterior density function for the parameters after the sample is proportional to the likelihood times the prior information. This way we can update our prior information that is modified by the sample information and attainment of the posterior density can be viewed as the end point of any scientific investigation.

Anyone who is familiar with the Bayesian approach is familiar with the everlasting arguments concerning the proper philosophical and probability foundations, pros and cons of this approach with respect to classical econometrics and difficulty to elicit prior distributions in most empirical analyses. These discussions are intellectually stimulating, but of little interest to practitioners. Therefore I prefer a more pragmatic approach and present an example where bayesian methods are the only meaningful solution to derive inferences about quantity of interest such as regional elasticity of substitutions. We know from previous research that interval of confidence can be obtained only with a bayesian approach (Gallant and Monham 1985) while all the neoclassical properties can be imposed only with bayesian a priori restrictions (Barnett, Geweke and Wolf 1991a, 1991b, Chalfant and Wallace 1991, Chalfant, Gray and White 1991). Then a bayesian transformation from prior to posterior knowledge must be adopted in our regional analysis and the very point is to adopt a reasonable prior. In general we have two choices. First we can claim to know to have no information, but neoclassical properties. In this case our prior is a Jeffrey's non informative or diffuse prior that satisfies neoclassical constraints. As we see below, in our standard linear regression model, this is to assume:

$$f\left(\beta,\sigma\right) \propto \frac{I\left(\beta,\breve{y}\right)}{\sigma}$$

where $I\left(\beta,\tilde{y}\right)$ is an indicator function equal to one when neoclassical constraints are satisfied and zero otherwise.

The second possibility is to form a proper prior that expresses our a priori knowledge about the phenomenon under investigation. I claim that any empirical analysis in regional science can be cast with a proper prior since we always have data at the national lavel. Therefore we can elicit a convenient prior derived from national data for the same parameters in which we are interested. Formally in the general linear model we can consider:

$$f\left(\beta,\sigma\right) \propto I(\beta,\breve{y})\, f_N\left(\beta / \sigma\right) f_{IG}(\sigma)$$

where prior distributions are simply formed using the same model with previous or contemporaneous national data. However this procedure is not completely automatic, since we can always monitor these distributions in a appropriate way. If for instance the regional data belong to an "important" region, whose share is relevant, we can give a lot of weight to this prior. On the other hand for marginal regions we can choose to make this prior distribution more diffuse simply controlling some a priori distribution parameters. Therefore this approach is flexible enough to suit regional science practitioners without resorting to complex and demanding aggregation theories.

Finally I would like to stress that bayesian econometrics appear to be the only feasible solution if we consider the huge variety of phenomena in regional science, as it doesn't pretend to discover the "true" model or the "true" data generating process. Perhaps in regional science we are more interested to study interregional differences and explain for instance why certain regions grow and others don't. When we compare regions we are better off if we specify a general common model within which we can

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[1] Classical refrence are Zellner (1971), Press (1972), Box and Tsiao (1973) while updated text are O'Hagan (1995), Bernardo and Smith (1994).

differentiate patterns. This task cannot be accomplish with the standard approach since "the fundamental difficulty for classical inference is that it deduces what should be observed as the sample increases if the model is correctly specified. By contrast, the objective of the investigator is to modify his view of the world conditional on a given data set, and prior information which includes the specification of the model" (Geweke 1988 p. 161).

The paper is organized as follows. In section 2, I briefly review the well known translog cost function. In section 3 I outline the standard bayesian approach for the general linear model with exchangeable observations. Regular regional elasticity distributions are analyzed with a Monte Carlo Composition method that can be used to determine the posterior probability of inequality constraints (monotonicity and concavity). Empirical results for the Italian macro regions are presented in section 4. Finally conclusions and directions for future research are discussed in the last section.

2. A generalized translog cost function

Consider a standard generalized transcendental logarithmic (translog):

$$\begin{aligned} ln\,C(p,q,t) = \alpha_0 + \alpha_Y\, ln\,q + + \frac{1}{2}\gamma_{YY}(ln\,q)^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_{ij}\, ln\,p_i\, ln\,p_j + \\ + \sum_{i=1}^{N}\gamma_{Yi}\, ln\,q\;\; ln\,p_i + \delta_T\, t + \frac{1}{2}\delta_{TT}\, t^2 + \delta_{TY}\, t\;\, ln\,q + \sum_{i=1}^{N}\delta_{Ti}\, t\;\, ln\,p_i \end{aligned} \qquad (2.1)$$

where q is total output, p a vector of non-negative prices and t is a time index. Cost shares are provided by Shepard Lemma:

$$y_i = \alpha_i + \sum_{i=1}^{N}\gamma_{ij}\, ln\,p_j + \gamma_{Yi}\, ln\,Y + \delta_{Ti}\, t \qquad \text{i} = 1,\dots,N \qquad (2.2)$$

It is known that a well behaved cost function and its share systems must satisfy several properties. Linear homogeneity in p can be globally imposed assuming:

$$\sum_i \alpha_i = 1;\; \sum_i \gamma_{ij} = \sum_j \gamma_{ij} = \sum_i \gamma_{Yi} = \sum_i \delta_{Ti} = 0\;. \qquad (2.3)$$

while symmetry of the cross price demand responses is satisfied if $\gamma_{ij} = \gamma_{ji}$ for all i,j. If the translog is conceived as a Taylor approximation to a generic cost function in $t=0$, $p_i=q=1$ for all i, by Young theorem, symmetry is a maintained hypothesis. Otherwise (2.2) is an exact cost function whose properties can be tested or assumed (Jorgenson

1986). Unfortunately there is no simple linear combination of parameter that guarantees concavity. This has usually been "tested" checking whether the substitution elasticities matrix is semidefinite negative for each actual share. However concavity can be imposed via Cholesky decomposition, as suggested by Lau (1978)[2]. Since the matrix of the share elasticities $\Omega = \left[\gamma_{ij}\right]$ is symmetric, than it is possible to represent it in terms of its Cholesky factorization $\Omega = T'DT$, where T is a unit lower triangular matrix and D is a diagonal matrix. Symmetry and product exhaustion provide conditions under which there exists a one to one transformation between the elements of ###Ω and the elements of T and D. Then the matrix of share elasticities is semidefinite negative if and only if the first *N*-1 diagonal elements of D are nonpositive. This procedure has been first applied by Jorgenson and Fraumeni (1981), but it has been shown by Diewert and Wales (1987) that it destroys the second order flexibility properties, since it implicitly assumes "restrictions on own and cross elasticities that may be *a priori* completely unacceptable ... the use of the Jorgenson-Fraumeni procedure for imposing concavity will lead to estimated input substitution matrices which are in some sense "too negative semidefinite"; i.e. the degree of input substitutability will tend to be biased in an upward direction" (Diewert and Wales 1987, p. 48). Below we follow the bayesian approach to impose locally concavity and monotonicity that doesn't suffer this drawback.

Parameters and their transformations provide information about the technology. In this paper we focus on price and substitution elasticity. Price elasticities can be easily obtained by Allen's definition of elasticities of substitution:

$$\sigma_{ij} = \frac{C\,(\partial^2 C / \partial p_i \partial p_j)}{\partial C / \partial p_i \;\; \partial C / \partial p_j} = \frac{\varepsilon_{ij}}{y_j} = \frac{\gamma_{ij}}{y_i y_j} + 1\,, \qquad \text{for } i \neq j \tag{2.4}$$

$$\sigma_{ii} = \frac{\varepsilon_{ij}}{y_i} = 1 - \frac{1}{y_i} + \frac{\gamma_{ij}}{y_i^2}\,, \tag{2.5}$$

while price elasticities are:

$$\varepsilon_{ij} = y_j \sigma_{ij} \tag{2.6}$$

Since we want to keep the analysis as simple as possible I choose a value added translog cost function that generates the following labor share equation:

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[2] A complete discussion about concavity can be found in Morey (1986).

$$y_t = \alpha_L + \gamma_{LL} \ln(p_{Lt} / p_{Kt}) + \gamma_{YL} \ln Y_t + \delta_{TL}\ t_t + \varepsilon_t, \qquad t = 1,\cdots T \qquad (2.7)$$

where y_t is the t-th observation of labor share and $\varepsilon_t \sim N(0,\sigma^2)$ is an homoschedastich error.

3. Statistical analysis

Following Geweke (1986) we assume standard linear regression[3] with a proper prior that presupposes a well behaved cost function:

$$f(\beta,\sigma) \propto I(\beta,\tilde{y})\, f_N(\beta/\sigma)\, f_{IG}(\sigma) \qquad (3.1)$$

since $I(\beta,\tilde{y})$ is an indicator function equal to one when inequality constraints are satisfied (i.e. when posterior distributions don't violate concavity and monotonicity). We stress the difference with Barnett, Geweke and Wolf (1991a), (1991b) or Chalfant, Gray and White (1991) who consider the indicator function depending on parameters only, while it's well known that concavity requires that substitution matrix is negative semidefinite[4]. This in turn constraint parameter and predictive to lie into a neoclassical regular space. Therefore the posterior is no longer a normal gamma inverse distribution since:

$$f(\beta,\sigma / y) \propto I(\beta,\tilde{y})\sigma^{-k} \exp\left\{-(\beta-b)'(X'X)(\beta-b)/ 2\sigma^2\right\}\sigma^{-(\nu+1)} \exp\left\{\frac{\nu s^2}{2\sigma^2}\right\}$$

(3.2)

where

$$b = (X'X)^{-1}X'y, \quad \nu s^2 = y'y - b'(X'X)\,b,\ \nu = T-k\,. \qquad (3.3)$$

Actually the predictive density function $f(\hat{y}/y, \hat{X})$, with $\tilde{X}:(\mathrm{q}\times \mathrm{k})$, and the β/y are not multi-t Student distribution with ν degrees of freedom as without inequality constraints (Zellner 1971). Within this framework sampling theory methods cannot be adopted due to the absence of any distribution theory and bayesian methods must be

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[3] More precisely assume $z_i = \{y_i, x_i'\}'$ exchangable[3]. Then, conditional on $\theta \in \Theta$, z_i/θ are i.i.d. Let the parameter vector be decomposed into two separate subvectors $\theta' = [\varphi', \psi']'$, where $\varphi = (\beta', \sigma)'$ and assume the hypothesis of bayesian cut (see Florens and Mouchart (1985)).

used. Actually maximum likelihood estimator of β is equal to b if either the constraints are not binding or β lies on the boundary of $I(\beta,\tilde{y})$. Moreover it can be shown that the distribution of the maximum likelihood estimator depends on the true value of β, and is not admissable as an estimator (Judge et al. 1985). Then the "computational coincidence of sampling and posterior distributions does not extend to inequality constrained linear regression" (Geweke 1986: p.128). For these reasons a bayesian approach must be preferred, even if we have to apply not standard computational procedures as Monte Carlo integration, since $I(\beta,\tilde{y})$ is an indicator function and analytic integration is not feasible. Our task is to calculate the following integral with inequality constraints:

$$E\left[g(\beta,\tilde{y})\right]=\frac{\int g(\beta,\tilde{y})\; l(\varphi / y)p(\varphi)\,d\varphi}{\int l(\varphi / y)p(\varphi)\,d\varphi}<+\infty \tag{3.4}$$

where $\varphi=(\beta',\sigma)'$. Analitical solutions are not feasible, but a Monte Carlo composition procedure is straightforward since:

$$f_R(\tilde{y},\beta,\sigma / y,\tilde{x})=I(\beta,\tilde{y} / \tilde{x})\,f_N(\tilde{y} / y,\beta,\sigma,\tilde{x})\,f_N(\beta / y,\sigma)\,f_{IG}(\sigma / y) \tag{3.5}$$

where $I(\beta,\tilde{y} / \tilde{x})$ is the indicator function evaluated at point. Then Monte Carlo method approximates any function over parameters and predictive with:

$$\hat{E}\left[g(\beta,\tilde{y}) / y,\tilde{x}\right]=\frac{\sum_{i=1}^{N} g(\beta_i,\tilde{y}_i)f_N(\tilde{y}_i / y,\beta_i,\sigma_i,\tilde{x})f_N(\beta_i / y,\sigma_i)\,f_{IG}(\sigma_i / y)I(\beta_i,\tilde{y}_i / \tilde{x})}{\sum_{i=1}^{N} f_N(\tilde{y}_i / y,\beta_i,\sigma_i,\tilde{x})f_N(\beta_i / y,\sigma_i)\,f_{IG}(\sigma_i / y)I(\beta_i,\tilde{y}_i / \tilde{x})} \tag{3.6}$$

where $(\beta_1,\sigma_1,\tilde{y}_1),....,(\beta_N,\sigma_N,\tilde{y}_N)$ are i.i.d. draws from the posterior distribution. By the Law of Large Numbers:

$$\hat{E}\left[g(\beta,\tilde{y}) / y,\tilde{x}\right]\xrightarrow{a.s.}E\left[g(\beta,\tilde{y})y,\tilde{x}\right] \tag{3.7}$$

while the estimated Monte Carlo standard error of $\hat{E}\left[\hat{g}(\varphi)\right]$ is:

[4] This implies that elasticities must satisfy the following conditions: $\sigma_{LL}\le 0$, $\sigma_{KK}\le 0$, $\sigma_{LL}\,\sigma_{KK}\ge 2\,\sigma_{KL}\ge 0$ (where the last inequality obtained by $\sum_{j=1}^{n}\varepsilon_{ij}=\sum_{j=1}^{n}S_j\;\sigma_{ij}=0$).

$$\frac{1}{\sqrt{N}}\sqrt{\frac{\sum_{i=1}^{N}\left\{\frac{\sum_{i=1}^{N} g(\beta_i, y_i)\, l(\varphi_i / y) p(\varphi_i)}{\sum_{i=1}^{N} l(\varphi_i / y) p(\varphi_i)} - \hat{E}\left[g(\beta_i, y_i)\right]\right\}^2}{N-1}} \tag{3.8}$$

Using this Monte Carlo composition method we get an empirical i.i.d. sample of the joint density over parameters and predicted with all the neoclassical restrictions. Furthermore this large sample can be used to approximate the posterior probability that the properties hold.[5] I can apply Monte Carlo integration as shown above to derive:

$$\begin{aligned} P\left[(\beta,\tilde{y}) \in (\Phi\times\Omega)_R / y\right] &= \int_{(\Phi\times\Omega)_R} f(\tilde{y},\beta / y,\tilde{x})\, d\beta\, d\tilde{y} \\ &= \int_{\Phi\times\Omega} I(\beta,\tilde{y}) f(\tilde{y},\beta / y,\tilde{x})\, d\beta\, d\tilde{y} \\ &\cong \frac{1}{N}\sum_{i=1}^{N} I(\beta,\tilde{y}_i) \end{aligned} \tag{3.9}$$

as suggested also by Chalfant and Wallace (1992) for Monte Carlo integration with importance sampling. This is also equal to (approximate) Posterior odds since obviously $P\left[(\beta,\tilde{y}) \in (\Phi\times\Omega) / y\right] = 1$. Using (2.35) we can state the condition that must be satisfied to accept concavity and monotonicity in a decision theoretic framework with piecewise continuous loss function. Call l_R is the loss incurred accepting incorrectly the restrictions and l_U is the loss if we reject incorrectly the restrictions.[6] Then an optimal decision minimizes the expected loss, hence we reject the restrictions if:

$$P\left[(\beta,\tilde{y}) \in (\Phi\times\Omega)_R / y\right] < \frac{l_R}{l_R + l_U} \tag{3.10}$$

As $0 \le P\left[(\beta,\tilde{y}) \in (\Phi\times\Omega)_R / y\right] \le 1$, loss function dictates the critical value for which restrictions hold. If we assume a symmetric loss function, i.e. $l_R = l_U$, we accept the restrictions if the posterior probability is larger than 0.5.

4. Data and empirical findings

[5] Note that restrictions are imposed at a point. However the approach cen be straightfowardly extended to impose them on a lattice.
[6] For sake of semplicity I have assumed that losses are independent on parameter and predicted.

The model is applied to the metal product sector concerning the Italian main macroregions (i.e. North, Center and South) over the years 1980-89. The complete data set concerns 20 Italian regions and is representative of a standard situation in (Italian) regional economics, since regional time series are usually extremely short. Labor comprehends both employed and self-employed workers, whose wage is assumed to be equal within each region. Capital stock data are not available for the eighties at the regional level, therefore it as been computed as a residual from constant price valued added minus constant wage payroll. Even if the residual approach to build the data set is very questionable, stocks and relative price are consistent with national patterns and other regional information such as regional investment.[7] Moreover I utilize an enlarged definition of capital as it can include factors that are not compatible with the neoclassical theory outlined above, such as market power. These caveats should be kept in mind when reviewing regional elasticitities.

The first step of our analysis is to adopt a prior. I embrace both a diffuse (noninformative) prior and a proper one. A Jeffrey's prior can be used if we claim not to have any a priori knowledge about parameters and has been frequently addressed as a reductive attempt to be more "objective". A proper prior can be easily derived from national data for the previous decade. Moreover we can also assume that the error variances are different in the two sets (Zellner 1971, Drèze 1977) and obtain a poly t 2-0 model or we can accommodate degrees of freedom to obtain very flat prior distributions[8]. Posterior moments are provided in Table 1 with moments for the proper prior. As obvious there is a greater precision with the informative prior.

[7] The value-added approach to estimate cost of capital (Cost of capital = value added - payroll) was criticized since it includes more than cost of reproducible capital, as working capital, land and so on. It was suggested (see also Christensen and Jorgenson (1969), Berndt and Wood (1975)) that a service price approach is better suited since it allows to directly figure out the cost of reproducible capital alone. This can explain the large difference in U.S. manufactoring elasticities in earlier studies, that "could in part be traceable to the fact that two quite different types of capital inputs are involved and these two form of capital, physical capital and working capital, behave in quite different ways" (Field and Grebenstein (1979: p. 207). However lack of data on regional physical capital and the extreme unreliability of the procedure adopted to construct rental price series for the Italian regions had forced to follow the value added approach

[8] In this analysis degrees of freedom are equal to 6, allowing for quite smooth distributions, but these can be decreased further, since only second order moments are required to have well defined prior distributions. For sake of simplicity I have implemented the most straightforward statistical model, but the bayesian approach is flexible enough to allow any modification about prior beliefs with a larger computational cost.

	INFORMATIVE PRIOR				DIFFUSE PRIOR		
	NORTH	CENTER	SOUTH	PRIOR	NORTH	CENTER	SOUTH
α_L	0.67734	0.65924	0.64535	0.71751	0.68055	0.65611	0.63901
st. err.	0.00278	0.00234	0.00293	0.01158	0.00271	0.00346	0.00380
γ_{LL}	0.19238	0.20726	0.21909	0.18793	0.25187	0.27113	0.30052
st. err.	0.02470	0.01908	0.02447	0.01283	0.07895	0.06619	0.05407
γ###$_{YL}$	-0.07710	-0.06902	-0.05222	-0.18234	-0.09755	-0.06843	-0.06003
st. err.	0.01820	0.01531	0.01742	0.02417	0.05927	0.06806	0.05447
δ###$_{TL}$	-0.00582	-0.00826	-0.01077	0.00364	-0.00968	-0.01222	-0.01350
st. err.	0.00131	0.00123	0.00135	0.00241	0.00439	0.00463	0.00322
σ###	0.00834	0.00660	0.00772	0.00351	0.00586	0.00620	0.00557
st. err.	0.00159	0.00126	0.00147	0.00127	0.00213	0.00225	0.00202

with concavity imposed

δ###$_{TL}$	-0.00558	-0.00779	-0.01055		0.00522	-0.00599	-0.00791
st. err.	0.00125	0.00110	0.00119		0.00308	0.00400	0.00307

TABLE 1 - Parameter Characteristics

As I said above parameter distributions are nor very interesting per sè, perhaps with the exception of the one relative to technical change. In Fig. B technical change marginal densities in South are plotted using a diffuse or a proper prior with (solid line) and without (dotted) concavity and monotonicity.

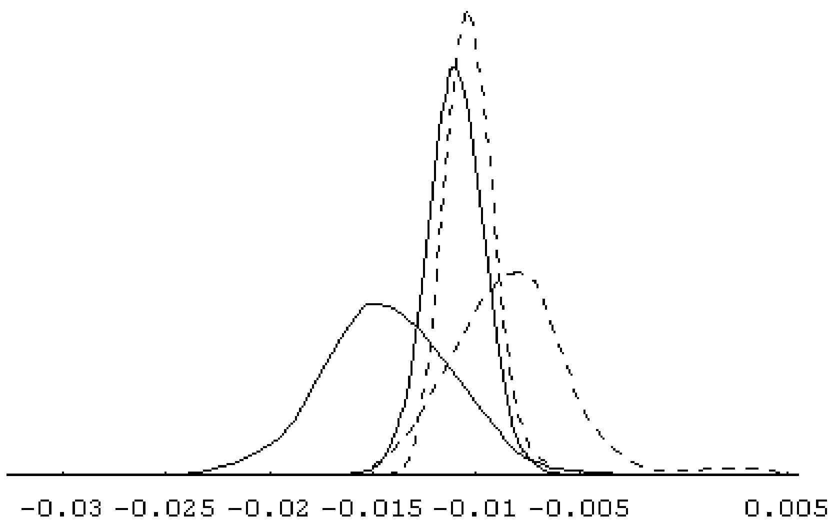

Fig. B   Technical change South

As we can see from Table 1 posterior expected values and standard errors with inequality constraints are very different with a diffuse prior. However in both cases it's evident that technical change has been labor saving (and capital using) in the eighties, since only a negligible part of the distribution with a diffuse prior covers positive values, even if our prior is mostly positive. The standard approach that estimate with maximum likelihood methods neglecting to impose concavity can be very misleading since a 95% Highest Posterior Density cover a large region that is not consistent with neoclassical theory.

Now let's consider more useful quantities as substitution and price elasticities. As a point of reference I choose the last year of the data set (1989). Results are reported in Table 2-4 and densities for price elasticities are plotted in Fig. C - E[9], while substitution elasticities are presented in Fig. F - H. We know that conditional on actual shares elasticities are distributed as univariate t Student. Therefore we can provide exact first moments that are shown in the following table:

[9] Recall that $\varepsilon_{KK} = \varepsilon_{KL}$ and $\varepsilon_{LL} = \varepsilon_{LK}$.

	INFORMATIVE PRIOR			DIFFUSE PRIOR		
	NORTH	CENTER	SOUTH	NORTH	CENTER	SOUTH
σ###$_{KL}$	0.14255	0.05044	0.03586	-0.12257	-0.24246	-0.32245
st. err.	0.11011	0.08743	0.10769	0.35188	0.30326	0.23794
σ###$_{KK}$	-0.27686	-0.10627	-0.06685	0.23805	0.51019	0.60109
st. err.	0.21385	0.18419	0.20075	0.44355	0.63890	0.44355
σ###$_{LL}$	-0.07340	-0.02394	-0.01924	0.06311	0.11494	0.17298
st. err.	0.05670	0.04150	0.05777	0.18118	0.14394	0.12764
ε###$_{KL}$	0.09410	0.03420	0.02334	0.08091	0.16422	0.20987
st. err.	0.07268	0.05929	0.07009	0.23228	0.20565	0.15487
ε###$_{LK}$	0.04845	0.01624	0.01252	0.04166	0.07795	0.11258
st. err.	0.03743	0.02814	0.03760	0.11960	0.09761	0.08308
ε###$_{KK}$	-0.09410	-0.03420	-0.02334	0.08091	0.16422	0.20987
st. err.	0.07273	0.05929	0.07009	0.23228	0.20565	0.15487
ε###$_{LL}$	-0.04845	-0.01624	-0.01252	0.04166	0.07795	0.11258
st. err.	0.03743	0.02814	0.03760	0.11960	0.09761	0.08308

TABLE 2 - Elasticities conditional on 1989 labor share

It’s striking to note that all the expected values for price and substitution elasticities with a diffuse prior have the "wrong" sign. Expected own elasticities are positive and cross elasticities are negative, against neoclassical theory. However, using the usual sampling theory language, we should immediately add that we cannot reject the hypothesis that they are not significantly different from zero. It’s not clear how useful is such a statement, but, in our framework, we can easily figure out the probability that elasticities are greater (or lower) than zero. In the same fashion we can calculate posterior odds (or Bayes Factor, if we assign equal prior probabilities to both hypotheses) in favor of the (marginal) regularity condition, as discussed above. At any rate Bayes Factor with a symmetric distribution are always against regularity (less than one) since expected values have the "wrong" sign. Therefore they are not presented here, and visual inspection of such distributions (the dotted ones) depicted in Fig. B - H is even more helpful to understand the point. The opposite holds with a proper prior. In this case all the expected values have the "correct" sign, but some variances are still quite large (even if three times lower than with a diffuse prior). In South there are still large regions of theoretical irregularity, as we can see from Fig. G.

Nonetheless this analysis is conceptually anomalous, since it is conditional on actual factor share. This is not very meaningful, as in our translog model, we should condition on the triplet *(Y,t,w/r)*. Moreover marginal distributions don't fully account for regularity conditions, that must be imposed at the same time on all the elasticities. These tasks are accomplished in two steps as discussed above. First we perform a Monte Carlo composition method to get a sample of 10.000 observations from the joint distributions of parameters and predicted. Then we use this sample to derive elasticities conditional on *(Y,t,w/r)* in 1989. Subsequently we impose concavity and plot the resulting distributions. All these are plotted together to check different behaviors. Distributions conditional on labor shares are plotted with a dotted line, while I have plotted with a solid line those conditional on *(Y,t,w/r)* with and without imposing concavity.

	INFORMATIVE PRIOR			DIFFUSE PRIOR		
	NORTH	CENTER	SOUTH	NORTH	CENTER	SOUTH
σ###$_{KL}$	0.13841	0.05088	0.03848	-0.11992	-0.24101	-0.31822
st. err.	0.11212	0.08993	0.10634	0.35575	0.31708	0.23709
σ###$_{KK}$	-0.27205	-0.10575	-0.07020	0.23032	0.50949	0.58942
st. err.	0.22055	0.18890	0.19623	0.67920	0.67211	0.44091
σ###$_{LL}$	-0.07054	-0.02450	-0.02112	0.06250	0.11416	0.17195
st. err.	0.05737	0.04297	0.05783	0.18763	0.15064	0.12808
ε###$_{KL}$	0.09196	0.03438	0.02472	-0.07884	-0.16357	-0.20661
st. err.	0.07438	0.06050	0.06929	0.23317	0.21520	0.15408
ε###$_{LK}$	0.04689	0.01655	0.01356	-0.04108	-0.07744	-0.11161
st. err.	0.03798	0.02889	0.03764	0.12271	0.10201	0.08310
ε###$_{KK}$	-0.09196	-0.03438	-0.02472	0.07884	0.16356	0.20661
st. err.	0.07438	0.06050	0.06929	0.23317	0.21520	0.15408
ε###$_{LL}$	-0.04689	-0.01655	-0.01356	0.04107	0.07744	0.11161
st. err.	0.03798	0.02889	0.03764	0.12271	0.10201	0.08310

TABLE 3 - Elasticities conditional on 1989 ($Y,t,w/r$) without inequality constraints

A few comments are worthwhile. As shown in Table 2 and 3, expected values conditional on labor share and on *($Y,t,w/r$)* without imposing concavity are very similar. Therefore we can conclude that the standard translog model predict quite well in terms of elasticities. Very different is the situation when we impose concavity. Results for expected values and standard deviations are tabulated in Table 4.

	INFORMATIVE PRIOR			DIFFUSE PRIOR		
	NORTH	CENTER	SOUTH	NORTH	CENTER	SOUTH
σ###$_{KL}$	0.17028	0.09648	0.10434	0.257961	0.221143	0.15507
st. err.	0.09366	0.06717	0.07662	0.23078	0.23160	0.15097
σ###$_{KK}$	-0.33486	-0.20167	-0.19176	-0.48632	-0.46109	-0.28678
st. err.	0.18358	0.59734	0.14077	0.43966	0.48857	0.28159
σ###$_{LL}$	-0.08674	-0.04622	-0.05686	-0.13714	-0.10641	-0.08406
st. err.	0.05737	0.03249	0.04198	0.12373	0.11238	0.08179
ε###$_{KL}$	0.11312	0.06592	0.06819	0.16846	0.14935	0.100591
st. err.	0.06191	0.04525	0.04892	0.15077	0.15653	0.09812
ε###$_{LK}$	0.05764	0.03155	0.03713	0.08949	0.07178	0.05448
st. err.	0.03172	0.02181	0.02679	0.08030	0.07539	0.05296
ε###$_{KK}$	-0.11312	-0.06592	-0.06819	-0.16846	-0.14935	-0.10059
st. err.	0.06191	0.04525	0.04892	0.15077	0.15653	0.09812
ε###$_{LL}$	-0.05764	-0.03155	-0.03713	-0.08949	-0.07179	-0.05448
st. err.	0.03172	0.02181	0.02679	0.08030	0.07539	0.05296

TABLE 4 - Elasticities conditional on 1989 (Y,t,w/r) with inequality constraints

Moments are quite dissimilar with a diffuse prior. For instance the expected capital labor substitution elasticity for the Center is double than with a proper prior, but no general pattern can be easily detected. However, it is very interesting to notice how the posterior odds in favor of concavity (given by the ratio of accepted samples) varies with regions. With a proper prior, in North the probability is in line with earlier findings by Chalfant and Wallace (1992) and is approximately about 0.538, while it declines in Center (0.397) and further in South (0.357). This is even more striking when we adopt a diffuse prior. Odds are very against concavity in South (0.043) and Center (0.102), while in North it is equal to 0.213. We can notice how neoclassical theory deteriorates in Southern regions. This is coherent with our previous findings since elasticities densities mostly don't cover neoclassical space in South. From an economic point of view, we notice how elasticities are higher in North but still quite low and that technology in South is very close to a Leontief production function. I would like to remark that, contrary to Guilkey and Lovell (1981), even with substitution elasticities lower than 1/3, the translog model still produces a reasonable approximation to the unknown cost function in North. However it will be interesting to compare the behavior of this functional flexible form with different ones (such as Generalized Leontief, Box-Cox or Minflex Laurent) in further studies.

5. Conclusions

In this paper I have reviewed the well known approach to analyze production technologies with dual flexible cost function. In this study I have chosen the standard generalized translog cost function that had widespread applications in the empirical literature. This has been applied to the metal product sector of the main macro region in Italy (North, Center and South).

Since elasticity distributions are not known and approximate variances are often completely unreliable I have adopted a bayesian approach that can provide a sound analysis, even with a very small sample (10 observations), providing a proper prior. As I deal with Italian regional cost function, the informative prior has been naturally formed with national data from the previous decade. As a point of reference I have adopted a diffuse prior to compare both results. The bayesian approach has been performed in two steps. First I derive posterior distribution with homogeneity and symmetry that cab be derive in a straightforward way in our natural conjugate framework. Then I have adopted a Monte Carlo Composition method to approximate them through an i.i.d. empirical sample from the joint distribution over parameter and predictive. Only the replications that satisfy all the neoclassical properties are accepted and also used to derive the posterior odds ratios concerning concavity and monotonicity. Empirical results are roughly in line with previous findings and as expected. This is true if we adopt a proper prior, as, with a diffuse one, even if translog parameters are significant, elasticities display unplausible values. Moreover concavity is not acceptable in all the regions. Even with a proper prior, neoclassical theory deteriorates in South and can be hardly accepted.[10] In any case price and substitution elasticities between capital and labor are very low but not far away from previous findings.

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[10] Fiorito (1990) estimates a labor market model in the same macroregions and find unsound results in South, while Prosperetti and Varetto (1991) found larger inefficiencies in South suggesting that neoclassical models could be at stakes in this region.

References:

Barnett, W.A., Geweke J. and Wolfe, M. (1991a), "Seminonparametric Bayesian estimation of the asymptotically ideal production model", Journal of Econometrics, **49**, 5-50.

Barnett, W.A., Geweke J. and Wolfe, M. (1991b), "Seminonparametric Bayesian estimation of applied general equilibrium models", in Barnett, W.A., Cornet, B., Claude d'Aspemont, Gabsezewicz, J.and Mas-Colell, A., Equilibrium theory and applications, Cambridge Un. Press, Cambridge.

Bernardo, J. M. and Smith, A.M.F. (1994), Bayesian Statistics, Wiley, New York.

Berndt, E.R. and Wood, D. O. (1975), "Technology, prices and the derived demand for energy, Review of Economics and Statistics, **56**, 259-268.

Box, G. E. P. and Tsiao G. C. (1973), Bayesian Inference in Statistical Analysis, Addison-Wesley, Reading Mass.

Chalfant, J. A., Gray, R. S. and White, K. J. (1991), "Evaluating prior beliefs in a demand system: the case of meat demand in Canada", American Journal of Agricultural Economics, **73**, 476-490.

Chalfant, J. A. and Wallace, N. E. (1992), "Bayesian analysis and regularity conditions on flexible functional forms: application to the U.S. motor carrier industry", in Griffiths, W.E, Hill, R.C., Lütkepohl, H. and Bock, M.E. (eds.), Readings in econometrics theory and practice, Elsevier Science Publishers, North Holland, New York, 159-175.

Christensen, L. R. and Jorgenson, D. W. (1969), "The measurament of U.S. real capital input, 1929-1967", Review of Income and Wealth, serie 15, 293-320.

Diewert, W. E. and Wales, T. J. (1987), "Flexible functional forms and global curvature conditions", Econometrica, **55**, 43-68.

Drèze, J. H. (1977), "Bayesian regression analysis using poly-t densities", Journal of Econometrics, **6**, 329-354.

Field, B. C. and Grebenstein, C. (1979), "Capital-energye substitution in U.S. manufacturing", Review of Economics and Statistics, **59**, 207-211.

Fiorito, R. (1990), "Tendenze del mercato del lavoro del Nord, Centro e Mezzogiorno: la disaggregazione di un modello aggregato", in Cimoli M., Musu I. (eds.), Cambiamento strutturale ed asimmetrie nell'economia italiana, Franco Angeli, Milano.

Florens, J.-P. and Mouchart, M. (1992), "Bayesian testing and testing bayesian", CORE discussion paper n 9220, Un. Catholique de Luovain, Louvain La Nueve.

Gallant, A. R. and Monaham, J. F. (1985), "Explicitely infinite-dimensional bayesian analysis of production technologies", Journal of Econometrics, **25**, 171-201.

Geweke, J. (1986), "Exact Inference in the inequality constrained normal linear regression model", Journal of Applied econometrics, **1**, 127-141.

Guilkey, D.K., Knox Lovell, C.A. and Sickles, R. C. (1983), "A comparison of three flexible functional forms", International Economic Review, **24**, 591-616.

Jorgenson, D. W. (1986), "Econometric methods for modeling producer behavior", in Griliches, Z. and Intriligator , M D. (eds.), Handbook of Econometrics, vol. III, North Hooland, Amsterdam.

Jorgenson, D. W. and Fraumeni, B.M. (1981),"Relative prices and technical change", in Berndt, E. and Field, B. (eds), Modeling and measuring natural resources substitution, MIT Press, Cambridge, Mass.

Judge, G. G., Griffiths, W.E, Hill, R.C., Lütkepohl, H. and Lee, T. (1985), The theory and practice of econometrics, 2nd ed. Wiley & Sons, New York.

Lau, L. T. (1978), "Testing and imposing monotonicity, convexity and quasi-convexity" in Fuss, M. A. and McFadden, D., Production economics: a dual approach, vol. 2, North Holland, Amsterdam, 409-45.

Morey, E. R. (1986), "An introduction to checking, testing and imposing curvature properties: the true function and the estimated function", Canadian Journal of Economics, **19**, 207-35.

O'Hagan, A. (1994), Kendall's Advanced theory of Statistics, vol. 2b, Edward Arnold, London.

Prosperetti, L. and Varetto F. (1991), I differenziali di produttività Nord-Sud nel settore manifatturiero. Un'analisi microeconomica, Il Mulino, Bologna.

Zellner, A. (1971), An introduction to bayesian inference in econometrics, Wiley, New York.